Design and Analysis of Salmonid Tagging Studies in the Columbia Basin, Volume XVIII

Precision and Accuracy of the Transportation-to-Inriver (T/I) Ratio Estimator of Survival Benefits to Juvenile Salmonids Transported Around the Columbia River Basin Dams

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THE DESIGN AND ANALYSIS OF SALMONID TAGGING STUDIES IN THE COLUMBIA BASIN

VOLUME XVIII

Precision and Accuracy of the Transportation-to-Inriver (T/I) Ratio Estimator of Survival Benefits to Juvenile Salmonids Transported Around the Columbia River Basin Dams

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The Design and Analysis of Salmonid Tagging Studies in the Columbia Basin

Other Publications in this Series

Volume I: Skalski, J. R., J. A. Perez-Comas, R. L. Townsend, and J. Lady. 1998. Assessment of temporal trends in daily survival estimates of spring chinook, 1994-1996. Technical report submitted to BPA, Project 89-107-00, Contract DE-BI79-90BP02341. 24 pp. plus appendix.

Volume II: Newman, K. 1998. Estimating salmonid survival with combined PIT-CWT tagging. Technical report (DOE/BP-35885-11) to BPA, Project 91-051-00, Contract 87-BI-35885.

Volume III: Newman, K. 1998. Experiment designs and statistical models to estimate the effect of transportation on survival of Columbia River system salmonids. Technical report (DOE/BP-35885-11a) to BPA, Project 91-051-00, Contract 87-BI-35885.

Volume IV: Perez-Comas, J. A., and J. R. Skalski. Submitted. Preliminary assessment of the effects of pulsed flows on smolt migratory behavior. Technical report to BPA, Project 89-107-00, Contract DE-BI79-90BP02341.

Volume V: Perez-Comas, J. A., and J. R. Skalski. Submitted. Analysis of in-river growth for PIT-tagged spring chinook smolt. Technical report to BPA, Project 89-107-00, Contract DE-BI79-90BP02341.

Volume VI: Skalski, J. R., J. A. Perez-Comas, P. Westhagen, and S. G. Smith. 1998. Assessment of season-wild survival of Snake River yearling chinook salmon, 1994-1996. Technical report to BPA, Project 89-107-00, Contract DE-BI79-90BP02341. 23 pp. plus appendix.

Volume VII: Lowther, A. B., and J. R. Skalski. 1998. Monte-Carlo comparison of confidence interval procedures for estimating survival in a release-recapture study, with applications to Snake River salmonids. Technical report (DOE/BP-02341-5) to BPA, Project 89-107-00, Contract 90-BI-02341.

Volume VIII: Lowther, A. B., and J. R. Skalski. 1998. Improved survival and residualization estimates for fall chinook using release-recapture methods. Technical report (DOE/BP-02341-6) to BPA, Project 89-107-00, Contract 90-BI-02341.

Volume IX: Townsend, R. L., and J. R. Skalski. Submitted. A comparison of statistical methods of estimating treatment-control ratios (transportation benefit ratios), based on spring chinook salmon on the Columbia River, 1986-1988. Technical report to BPA, Project 91-051-00, Contract 87-BI-35885.

Volume X: Westhagen, P., and J. R. Skalski. 1998. Instructional guide to using program CaptHist to create SURPH files for survival analysis using PTAGIS data files. Technical report (DOE/BP-02341-4) to BPA, Project 89-107-00, Contract 90-BI-02341.

Volume XI: Skalski, J. R., R. L. Townsend, A. E. Giorgi, and J. R. Stevenson. Submitted. Recommendations on the design and analysis of radiotelemetry studies of salmonid smolts to estimate survival and passage efficiencies. Technical report to BPA, Project 89-107-00, Contract DE-BI79-90BP02341. 33 pp.

Volume XII: Ryding, K. E., and J. R. Skalski. 1999. A multinomial model for estimating ocean survival from salmonid coded wire-tag data. Technical report (DOE/BP-91572-3) to BPA, Project 91-051-00, Contract 96-BI-91572.

Volume XIII: Perez-Comas, J. A., and J. R. Skalski. 2000. Appraisal of system-wide survival estimation of Snake River yearling chinook salmon using PIT-tags recovered from Caspian tern and double-crested cormorant breeding colonies on Rice Island. Technical report to BPA, Project No. 8910700, Contract DE-BI79-90BP02341.

Volume XIV: Perez-Comas, J. A., and J. R. Skalski. 2000. Appraisal of the relationship between tag detection efficiency at Bonneville Dam and the precision in estuarine and marine survival estimates of returning pit-tagged chinook salmon. Technical report to BPA, Project No. 8910700, Contract DE-BI79-90BP02341.

Volume XV: Perez-Comas, J. A., and J. R. Skalski. 2000. Appraisal of the relationship between tag detection efficiency at Bonneville Dam and the precision in-river survival estimates of returning PIT-tagged chinook salmon. Technical report to BPA, Project No. 8910700, Contract DE-BI79-90BP02341.

Volume XVI: Skalski, J. R., and J. A. Perez-Comas. 2000. Alternative designs for future adult PIT-tag detection studies. Technical report to BPA, Project No. 8910700, Contract DEBI79-90BP02341.

Other Publications Related to this Series

Other related publications, reports and papers available through the professional literature or from the Bonneville Power Administration (BPA) Public Information Center - CKPS-1, P.O. Box 3621, Portland, OR 97208.

Lowther, A. B., and J. R. Skalski. 1998. A multinomial likelihood model for estimating survival probabilities and overwintering for fall chinook salmon using release-recapture methods. Journal of Agricultural, Biological, and Environmental Statistics 3: 223- 236.

Skalski, J. R. 1998. Estimating season-wide survival rates of outmigrating salmon smolt in the Snake River, Washington. Canadian Journal of Fisheries and Aquatic Sciences 55: 761-769.

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Smith, S. G., J. R. Skalski, J. R., J. W. Schlechte, A. Hoffmann, and V. Cassen. 1996. Introduction to SURPH.1 analysis of release-recapture data for survival studies. Technical report DOE/BP-02341-3) to BPA, Project 89-107-00, Contract 90-BI-02341.

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Smith, S. G., J. R. Skalski, J. R., J. W. Schlechte, A. Hoffmann, and V. Cassen. 1994. SURPH.1 Manual: Statistical survival analysis of fish and wildlife tagging studies. Technical report (DOE/BP-02341-2) to BPA, Project 89-107-00, Contract 90-BI-02341.

Dauble, D. D., J. R. Skalski, A. Hoffmann, and A. E. Giorgi. 1993. Evaluation and application of statistical methods for estimating smolt survival. Technical report (DOE/BP-62611-1) to BPA, Project 86-118-00, Contract 90-AI-62611; Project 89-107-00, Contract 90-BI-02341; and Project 91-051-00, Contract 87-BI-35885.

Skalski, J. R., A. Hoffmann, and S. G. Smith. 1993. Development of survival relationships using concomitant variables measured from individual smolt implanted with PIT-tags. Annual report 1990-1991 (DOE/BP-02341-1) to BPA, Project 89-107-00, Contract 90-BI-02341.

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Giorgi, A. E. 1990. Mortality of yearling chinook salmon prior to arrival at Lower Granite Dam on the Snake River. Technical report (DOE/BP-16570-1) to BPA, Project 91-051-00, Contract 87-BI-35885.

Preface

Project 199105100 was initiated in 1991 in response to the Endangered Species Act (ESA) and the subsequent 1994 Northwest Power Planning Council Fish and Wildlife Program (FWP) call for regional analytical methods for monitoring and evaluation. Primary objectives and management implications of this project include: (1) to assist in the development of improved monitoring capabilities, statistical methodologies to aid management in optimizing operational and fish passage strategies to maximize the protection and survival of listed, threatened, and endangered Snake River and Columbia River salmon populations; (2) to design better analysis tools for evaluation programs; and (3) to provide statistical support to the Bonneville Power Administration and the Northwest fisheries community.

All studies in the current series, the Design and Analysis of Tagging Studies in the Columbia Basin, were conducted to help maximize the amount of information that can be obtained from fish tagging studies for the purposes of monitoring fish survival throughout its life cycle. Volume IX of this series presents a statistical evaluation of methods for assessing the difference in smolt-to-adult returns (SARs) between transported smolts and inriver-migrating smolts. This report investigates the *transportation-to-inriver (T/I) ratio* (previously called the *transportation benefit ratio, TBR*), a commonly used measure for assessing the benefits of transporting juvenile salmon around dams in barges or trucks, relative to leaving them in the river to navigate the dams in their outmigration to sea. The T/I ratio is the ratio of transported to untransported SARs. This report describes the statistical properties of the T/I ratio using computer-intensive resampling techniques and analytical methods under different scenarios of survival and size of transported or inriver groups. Recommendations are provided on the preferred methods of estimating the T/I ratio and confidence interval construction.

Abstract

The accuracy and precision of the widely used transportation-to-inriver (T/I) ratio estimator are investigated using analytical and Monte Carlo methods. The T/I ratio is the ratio for smolt-to-adult (SAR) rates for transported and untransported salmonid smolts. Repeated simulations of a binomial likelihood model under varying values of adult return rate, sample size, and true T/I ratio to examine the distributional properties of alternative T/I ratios. A bias-corrected version of the T/I estimator was found to be less biased and to have smaller variance than the traditional estimator, under all possible values of adult return rate, sample size, and true T/I ratio. Although the bias of the original estimator is positive, the bias of the corrected estimator is slightly negative. Consequently, the traditional T/I ratio estimator has a greater chance of falsely identifying a transportation benefit effect from the bias-corrected estimator. An asymptotic lognormal 100 $(1-\alpha)$ confidence interval, constructed using the log of the bias-corrected estimator, is shown to have optimal coverage properties, compared to the asymptotic normal confidence interval, and has comparatively shorter interval length. An example using the bias-corrected estimator with an asymptotic lognormal confidence interval is provided, using PIT-tag release and return data from the 1995 and 1996 transportation experiments.

Relationships between precision of the bias-corrected estimator and sample size are investigated under different values of true T/I, adult return rate, and α -level. The variance of the T/I estimator under the scenario of the size of the inriver group estimated was also derived. Sample size requirements for precision of this estimator is are generally exorbitant.

Executive Summary

Objectives

1. To investigate the statistical properties of a common used estimator (the transportation-to-inriver, or T/I, ratio) for assessing the benefits of transporting smolts around Columbia River Basin dams versus leaving them inriver. Accuracy and precision were investigated using the delta method of approximating expectation and variance of an estimator.

2. To develop a statistical model of the prototypical transportation experiment with clearly stated assumptions involving two possible scenarios. One scenario describes the situation in which the control group size is known; the other describes the situation in which the control group size is unknown and must be estimated. The latter scenario arises when estimating the number of undetected fish passing through a hydroelectric project.

3. To perform Monte Carlo simulations to evaluate alternative T/I ratio estimators and confidence interval (CI) calculations.

4. To describe the precision of the T/I ratio estimator as a function of the number of fish in the transported and control groups, given particular values of adult return rate, true T/I ratio, and α -level.

5. To provide a numerical example of the T/I ratio estimator and CI calculations.

Methods

A likelihood model was developed for the juvenile salmonid transportation experiment and assumptions stated. Using the delta method, formulas for the mean, variance, and bias of the T/I ratio estimator were derived for two scenarios. In the first scenario, the inriver group size C was assumed known, and in the second, it was assumed unknown and must be estimated.

A bias-corrected version of the T/I estimator was developed and compared with the original estimator, using Monte Carlo simulation methods. The Monte Carlo methods consisted of simulating large numbers of the T/I ratio estimates under model assumptions, then computing sample means and variances which approximately equal the true expectation and variance of the estimators for large numbers of repetitions. By examining the behavior of expectation and variance under different survival and sample (release) size conditions, a determination was made as to which of the estimators was better, given the conditions.

Four alternative CI formulations were compared by simulating coverage probabilities of the T/I ratio under different values of adult return and release size, for true T/I values of 1.4 and 1.8. The four alternatives consisted of two each constructed from the original and the bias-corrected estimators, one asymptotic lognormal, and the other asymptotic normal.

Having a chosen a best estimator and best CI, absolute precision of that estimator was explored as a function of sample (release) size for different α -levels, adult return rates, and T/I ratio values.

An example from real life using PIT-tag data was provided.

Findings

Comparing the traditional T/I ratio estimator with a bias-corrected version of the T/I ratio revealed that under any regime of adult return rate, release size, and true T/I ratio, the bias-corrected version has less bias than the original T/I estimator. The character of the bias is different between the two estimators. Both estimators were found to have increased bias with decreasing adult return rate, larger values of T/I ratio, and smaller release sizes. The original estimator has positive bias, while the bias-corrected estimator has a negatively smaller bias. The practical implication is that the original estimator with its positive bias will, when adult return rates are small, produce an estimate of transportation benefit that is likely too high. The positive bias could lead to the incorrect conclusion that transported juveniles have higher adult return rates than juveniles left to migrate inriver. Using the bias-corrected estimator, on the other hand, there is a small chance the T/I ratio will underestimate the benefits of smolt transportation.

The bias-corrected estimator of the T/I ratio had smaller variance than the original estimator, particularly for small adult return rates, small sample (release) sizes, and large values of the true T/I ratio. For tag release sizes greater than 100,000, there was little difference in the variances of the two estimators.

Coverage probabilities were estimated for four formulations of 95% CIs for the T/I ratio. Coverage probability is the probability a CI will contain the true T/I ratio. The probabilities were estimated using Monte Carlo simulations under different return rates, T/I ratios, and sample sizes. The best coverage properties, i.e., nominal coverage of the true T/I ratio and minimal coverage of false T/I ratios, were found for asymptotically lognormal CIs constructed from the bias-corrected estimator.

Sample (release) sizes greater than 7500 are required to distinguish a T/I ratio of 1.4 from 1.0 with 95% confidence if the adult return rate of the control group is 0.01. If the control group adult return rate is 0.001, sample sizes grater than 75,000 are required; and for adult return rates of 0.0001, sample sizes of greater than 750,000 are required. The above sample sizes will suffice when the control group release size is known. If it is unknown and must be estimated with a coefficient of variation equal to 0.1, sample sizes greater than 325,000; 370,000; and 850,000 would be required to achieve the level of precision specified above, for adult return rates of 0.01, 0.001, and 0.0001, respectively.

An example data set, using PIT-tag data from the 1995 and 1996 transportation experiments, was used to compute the original and bias-corrected estimates for T/I ratio, an asymptotic normal 95% CI using the original estimator, and asymptotic lognormal 95% CI using the bias-corrected estimator. For the combined wild and hatchery data of 1996, the bias-corrected lognormal CI did not include 1.0, while the original estimator with an asymptotic normal CI did. Large differences between the original and bias-corrected estimates were observed when return rates were very small (approximately 0.0005).

Management Implications

Transportation is a central mitigation strategy of government programs to restore Columbia River Basin stocks to viable levels. Accurate assessment of benefits due to transportation are paramount to decisions about its continued practice. The statistical methods developed in this report should contribute to more accurate and precise information on transportation benefits and proper management decisions.

Table of Contents

List of Tables

List of Figures

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1.0 Introduction

Since 1991, eight evolutionarily significant units (ESUs) of Columbia River Basin salmon and steelhead (*Oncorhynchus* spp.) whose natal streams are upriver from Bonneville Dam have been listed as endangered or threatened under the Endangered Species Act (ESA 1973) (Table 1.1). The ESA mandates the National Marine Fisheries Service (NMFS) to design research agendas whose results will guide policy toward restoration of listed stocks to sustainably viable levels. In conjunction with the Bonneville Power Administration (BPA), US Bureau of Reclamation, and US Army Corps of Engineers (USACE), the NMFS's Northwest Region has overseen more than 25 years of research into how to achieve this restoration. The main focus of their freshwater research agenda was to mitigate effects of hydroelectric projects on salmon passage and survival. The two main approaches to mitigation have been reengineering the hydroprojects and transporting juvenile salmon around them. This report concerns the latter enterprise.

Table 1.1. Columbia River Basin stocks originating above Bonneville Dam listed as threatened or endangered under the Endangered Species Act since 1991.

Evolutionarily Significant Unit Listed	Listed Status	Year Listed
Snake River Fall-run Chinook Salmon	Threatened	1992
Snake River Spring/Summer-run Chinook Salmon	Threatened	1992
Upper Columbia River Spring-run Chinook Salmon	Endangered	1999
Columbia River Chum Salmon	Threatened	1999
Snake River Sockeye Salmon	Endangered	1991
Upper Columbia River Steelhead Trout	Endangered	1997
Snake River Basin Steelhead Trout	Threatened	1997
Middle Columbia River Steelhead Trout	Threatened	1999

From the NMFS website, **http://www.nwr.noaa.gov/1salmon/salmesa/pubs/1pgr.pdf**.

An estimated 15 to 20 million juvenile salmon and steelhead are transported from the Snake and Columbia rivers to below Bonneville Dam every year (Table 1.2) (http://www.nwd.usace.army.mil/ps/juvetran.htm) and tens to hundreds of thousands of wild and hatchery salmonid smolts are tagged with passive integrated transponder (PIT) tags or coded-wire-tags (CWT) so that transportation effects can be evaluated (Table 1.3) (Achord et al. 1992; Harmon et al. 1993, 1995, 1996; March et al. 1996, 1998). Juvenile bypass operations and juvenile transportation are the primary alternative mitigating strategies to dam-breaching. USACE research dollars alone spent on transportation have exceeded 20 million dollars since 1971, and transportation has accounted for approximately 7% of USACE's fish mitigation construction costs and 16% of fish mitigation operating costs since 1983, exceeding a hundred million dollars. Since 1983, nearly 20% of the USACE research budget went to study transportation. Future USACE transportation research, with an annual budget of $3 million (Adele Merchant, USACE, e-mail communication) will benefit from the slated deployment of adult PIT-detectors at Bonneville and potentially other dams and from the new 134.2-kHz ISO-based network of PIT-tag interrogation systems which replaced the old 400-kHz network and underwent final testing in 2000.

Table 1.2. Annual numbers of juvenile salmon and steelhead transported around Snake and Columbia river dams to below Bonneville Dam from 1982 to 1999. Prior to 1993, fish were transported from Lower Granite, Little Goose, and McNary dams. Since 1993, fish have also been transported from Lower Monumental Dam.

Year	Juveniles Transported (to the nearest thousand)
1982	5,813,000
1983	7,516,000
1984	887,000
1985	14,320,000
1986	13,209,000
1987	16,417,000
1988	19,574,000
1989	14,944,000
1990	21,030,000
1991	15,366,000
1992	17,317,000
1993	14,798,000
1994	16,713,000
1995	18,568,000
1996	11,164,000
1997	12,246,000
1998	18,407,000
1999	18,806,000
Total	265,077,000

Data compiled from Anderson et al. (2000).

Table 1.3. Summary of transportation/inriver ratios (T/I) as described in the NMFS April 2000 White Paper "Summary of research related to transportation of juvenile anadromous salmonids around Snake and Columbia river dams."

Species/Run	Release Location(s)	Truck/ Barge	Years	Range of T/I (Confidence Intervals)	No. of Studies	Summary of T/I Results	Average T_{SAR} (Percentage)	Comments
Yearling Chinook	Snake River dams	T	1968-1980	0.7 – 18.1	16	2 – T < I 2 – 0 adult return 6 – T significant > I 10 – Too few adults to test for significance	0.0 – 9.0	
Subyearling Chinook	McNary	T	1978-1983	2.3 – 10.1	6	6 – T significant > I		
Steelhead	Snake River dams	T	1970-1978	1.5 – 13.5	13	13 – T significant > I	0.4 – 4.7	
Steelhead	McNary	T	1978-1980	1.3 – 3.0	3	2 – T significant > I 1 – T > I		
Sockeye	Priest Rapids	T	1984-1988	0.55 – 4.23				No estimates of SAR because trapping efficiencies weren't measured. No analysis in 1987, 1988 because of low flow.
Yearling Chinook	Lower Granite	B	1977-1980	1978: 8.9 1979: 3.9	4	2 – 0 adult return (1977, 1980) 2 T significant > I	0.002 – 0.35	

Table 1.3. (Continued)

Species/Run	Release Location(s)	Truck/ Barge	Years	Range of T/I (Confidence Intervals)	No. of Studies	Summary of T/I Results	Average T_{SAR} (Percentage)	Comments
Yearling Chinook	Lower Granite	?	1986, 1989	1986: 1.6 (1.01-2.47) 1985: 2.4 (1.4-4.3)			0.32 (1986) 0.12 (1989)	No studies planned for 1987, 1988 due to low flow.
Yearling Chinook	Lower Granite	?	1995, 1996, 1998					1995 was first year tagged C fish were released directly into tailrace of dam rather than transported to Little Goose.
Yearling Chinook/Steelhead	Lower Granite	?	1999					
Yearling Chinook	McNary	B	1986-1988	1987: 1.6 (1.18-2.25) 1988: 1.6 (1.0-2.6)	5?	1 – 0 adult return 3 – CI included 1		
Subyearling Chinook	McNary	B	1977-1980, 1986-1989	1983: 2.9 1986: 2.8 (1.4-5.6) 1987: 3.5 (1.7, 7.1) 1988: 3.3 (1.3-9.4)	4 2	4 – T significant > I 2 – T significant > I	0.9-4.7% 1986: 1.3 1989: 0.6	
Steelhead	Snake River dams	B	1977-1980, 1986-1989	1977-80: 1.7-17.5 1986: 2.0 (2.4-2.7) 1989: 2.1 (1.3, 3.5)	2	2 – T significant > I		

Transportation studies require the use of some type of mark or tag which will allow for the tabulation of returning adults that were transported (T) as juveniles and for distinguishing them from returning adults that were not (C, control, of I, inriver). Prior to 1995, CWT were used exclusively for evaluating transportation experiments (NMFS 2000). These are small bits of wire implanted in smolts' nasal cartilage, and identify fish to a specific batch. In 1986, Lower Granite Dam was equipped with instrumentation to detect PIT-tags tags in returning adults (Prentice et al. 1990a, b, c). PIT-tags are injected into the body cavity and identify individual fish. The marking and handling of fish has been identified as a traumatic source of mortality in transported and control subjects (Mundy et al. 1994, US Fish and Wildlife Service 1993, NMFS 2000). After 1992, juvenile were anesthetized prior to handling and marking, and this continuing practice has lessened its deleterious effects (NMFS 2000).

Table 1.3 summarizes results of transportation experiments as reported in the NMFS White Paper (2000) entitled, A summary of research related to transportation of juvenile anadromous around Snake and Columbia river dams." Between 1968 and 1977, trucks were used exclusively to transport juvenile chinook, sockeye salmon, and steelhead trout around various Snake River dams and around McNary Dam located on the mainstem of the Columbia River just downriver from its confluence with the Snake. Currently, barges transport fish during April through June, and trucks are used before and after this period (NMFS 2000). This arrangement puts juvenile yearling chinook salmon and steelhead trout in barges for the most part (>95%) and juvenile subyearling chinook salmon in trucks. Questions still exist concerning the potential benefits of smolt transportation. Issues include delayed mortality of handled fish, changes in homing behavior, high mortality in transported fish, species differences in survival during and after transportation, high variability in river conditions within and between years which influence adult return rates in ways that are not well-understood, heterogeneity in experimental protocols, and lack of experimental replication.

Statistical methods of analyzing transportation experimental data have centered around the transportation-to-inriver survival ratio, T/I (also called transportation benefit ratio, TBR). The widely used estimator of T/I benefit is the empirical ratio of transported-to-inriver smolt-to-adult return (SAR) rates. This is the estimator used in Table 1.3. A T/I ratio greater than 1 is indicative of a transportation benefit, while a value less than 1 would suggest a detrimental effect due to transportation.

In the following chapters, we develop the statistical properties of the empirical T/I ratio and list assumptions for the likelihood model on which it is based. Statistical properties are investigated using asymptotic methods and Monte Carlo simulations. We also develop a bias-corrected estimator of T/I ratio and investigate its statistical properties. Optimal confidence interval coverage is also assessed. Precision of T/I ratios as a function of sample size will be described for different α -levels, adult return rates, and T/I ratio.

2.0 Statistical Methods

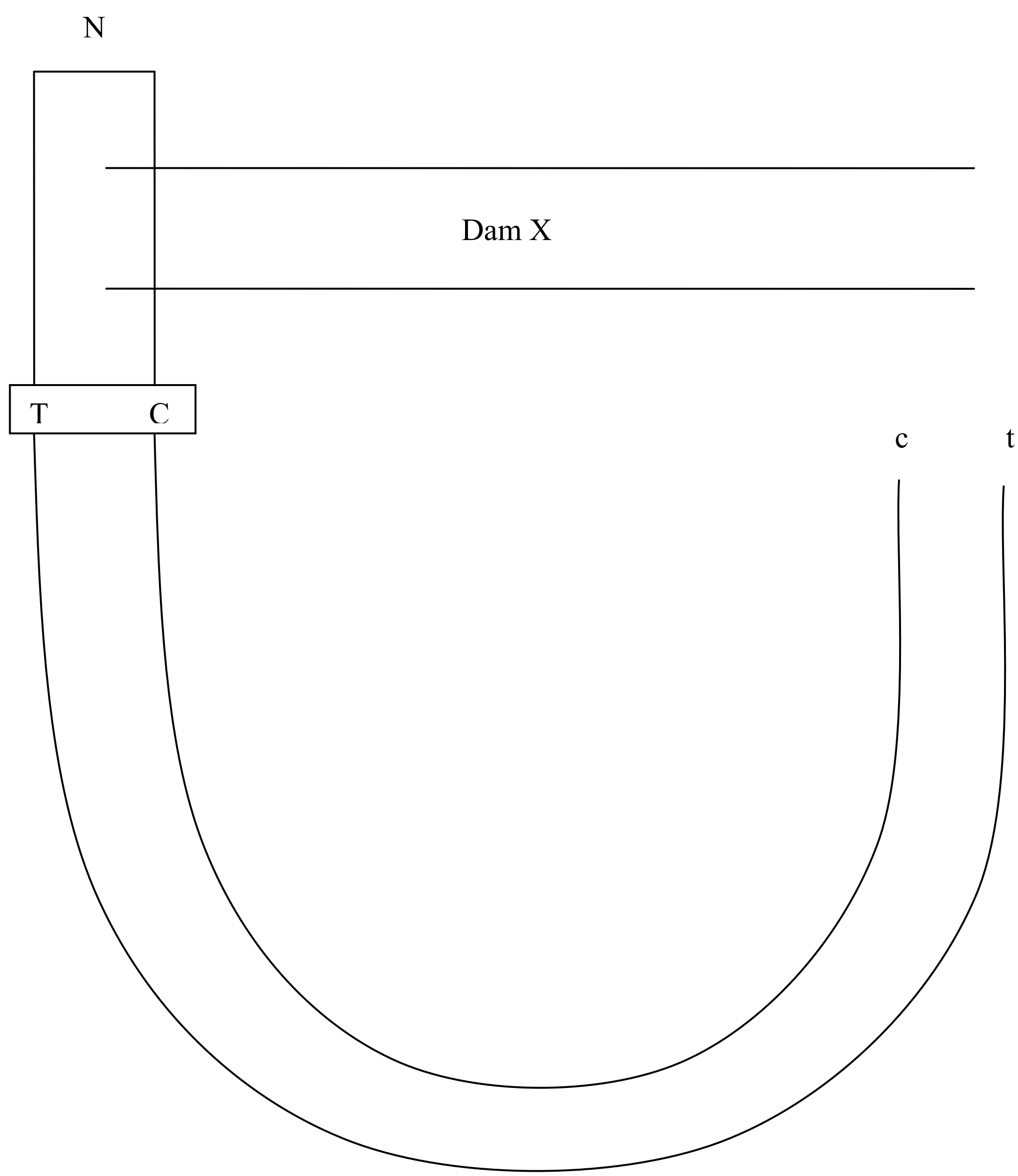
Increased survival due to juvenile transportation is demonstrated when the adult return rate of transported fish, S_T, is higher than the adult return rate of control fish, S_C, who outmigrate inriver. Figure 2.1 illustrates the basic transportation scenario in which N juvenile migrants are tagged at a site above a dam, T of these are transported around the dam, and $C = N - T$ are returned to the river to migrate downstream. The number of returning adults detected from the transported group is t and the number of returning adults detected from the control group is c. The ratio of the adult return rate of transported fish (t/T) to the adult return rate of control fish (c/C),

$$\hat{R} = \frac{\hat{S}_T}{\hat{S}_C} = \frac{\frac{t}{T}}{\frac{c}{C}} = \frac{tC}{cT}, \tag{2.1}$$

has been termed the transportation benefit ratio (TBR) or the ratio of transported-to-inriver (T/I) survivals.

Figure 2.1. Essential elements of transportation studies used to assess benefits to survival when the size of the untransported or control group is known. N fish are marked and divided into two groups; control (C) fish are left to migrate inriver while transported (T) fish are collected or barged to below Bonneville Dam. The number of recovered adults from the transported (T) and control (C) releases are t and c, respectively.

Scenario 1: T and C are known.



2.1 Likelihood Models

The assumptions used in the estimation of the T/I ratio are as follows:

1. Each smolt has an equal probability of surviving and returning as an adult within a release group.
2. Fates of all smolts are independent.
3. All smolts have equal probability (p) of detection upon return as an adult.
4. All smolt have equal survival below the downstream mixing zone for transported and control smolts.

To construct the likelihood function, the following parameters are defined:

p = probability an adult is detection upon return,

S_T = survival probability for transported smolt,

S_C = survival probability for inriver/control smolt,

T = number of transport smolt released,

t = number of transport smolt recovered as adults,

C = number of inriver/control smolt released,

c = number of inriver/control smolt recovered as adults.

Based on the assumptions (1-4), $t \sim Bin\left(T, S_T p\right)$ and $C \sim Bin\left(C, S_C p\right)$. In which case, the joint likelihood model for a transportation/inriver study can be written as, defining $R = S_T / S_C$,

$$L\left(R, S_C, p \middle| t, c, T, C\right) = \binom{T}{t}\left(RS_C p\right)^t \left(1 - RS_C p\right)^{T-t} \cdot \binom{C}{c}\left(S_C p\right)^c \left(1 - S_C p\right)^{C-c}$$

and further letting $\theta = S_C p$

$$L\left(R, \theta \middle| t, c, T, C\right) = \binom{T}{t}(R\theta)^t (1 - R\theta)^{T-t} \cdot \binom{C}{c}\theta^c (1-\theta)^{C-c}. \tag{2.2}$$

The maximum likelihood estimators for the model parameters are $\hat{\theta} = c / C$ and $\hat{R} = tC / Tc$.

2.2 Properties of $\hat{R}$

2.2.1 Variance of $\hat{R}$

The T/I ratio estimator, $\hat{R}$, has variance

$$Var\left(\hat{R}\right)=\left(\frac{C}{T}\right)^2 Var\left(\frac{t}{c}\right).$$

The variance of t/c can be approximated by the delta method (Seber 1982, see also Appendix A), in which case

$$Var\left(\hat{R}\right)\doteq R^2\left[\frac{1-R\theta}{TR\theta}+\frac{1-\theta}{C\theta}\right].$$

The variance can be estimated by the expression

$$\widehat{Var}\left(\hat{R}\right)=\left(\hat{R}\right)^2\left[\frac{1}{t}+\frac{1}{T}+\frac{1}{c}+\frac{1}{C}\right]. \tag{2.3}$$

2.2.2 Expected Value of $\hat{R}$ and its Bias

To the first-term Taylor series expansion

$$E\left(\hat{R}\right)\cong\frac{S_T}{S_C}. \tag{2.4}$$

However, the delta method allows for estimation of the higher order bias in $\hat{R}$, to the third-term of a Taylor series expansion (Appendix A)

$$\begin{aligned}\text{Bias }\left(\hat{R}\right)&=\hat{R}\left(\frac{1-\frac{c}{C}}{C\left(\frac{c}{C}\right)}\right)\\&=\hat{R}\left(\frac{1}{c}-\frac{1}{C}\right)\end{aligned} \tag{2.5}$$

which will always be greater than or equal to zero because $c\le C$, so

$$\frac{1}{c} \geq \frac{1}{C}.$$

2.3 Bias-Corrected $\hat{R}$

Bias (2.4) can be subtracted from (2.1) to produce a bias-corrected estimator of R,

$$\hat{R}_{BC} = \hat{R} - \hat{R}\left(\frac{1}{c} - \frac{1}{C}\right) = \hat{R}\left(1 - \frac{1}{c} + \frac{1}{C}\right). \tag{2.6}$$

The variance of $\hat{R}_{BC}$ can be approximated by the expression

$$\begin{aligned} Var\left(\hat{R}_{BC}\right) = \left(\frac{C}{T}\right)^2 & \left\{(TR\theta)^2 Var\left(\frac{1}{c} - \frac{1}{c^2} + \frac{1}{Cc}\right) + \left[E\left(\frac{1}{c} - \frac{1}{c^2} + \frac{1}{Cc}\right)\right]^2 \right. \\ & \left. \cdot TR\theta(1 - R\theta) + TR\theta(1 - R\theta) \cdot Var\left(\frac{1}{c} - \frac{1}{c^2} + \frac{1}{Cc}\right)\right\}. \end{aligned} \tag{2.7}$$

and estimated by the formula

$$\widehat{Var}\left(\hat{R}_{BC}\right) = \left(\frac{C}{T}\right)^2 \cdot \left[t^2\left(\frac{c(C-c)}{C}\right)\left(\frac{1}{c} + \frac{1}{c^2} - \frac{1}{Cc}\right)^2 + \left(\frac{t(T-t)}{T}\right)\left(\frac{c(C-c)}{C}\right)\left(\frac{-1}{c^2} + \frac{2}{c^3} - \frac{1}{Cc^2}\right)^2\right]. \tag{2.8}$$

2.4 Precision of $\hat{R}$ and $\hat{R}_{BC}$

In general, if ϕ is any parameter and $\hat{\phi}$ is an estimator of ϕ, the absolute error in estimation of ϕ defined as $\left|\hat{\phi} - \phi\right|$, which of course we desire to be small. Sampling precision can then be expressed as the desire for the absolute error $\left|\hat{\phi} - \phi\right|$ to be less than ε, with probability $1 - \alpha$. Or expressed more formally as

$$P\left(\left|\hat{\phi} - \phi\right| < \varepsilon\right) = 1 - \alpha \tag{2.9}$$

or

$$P\left(-\varepsilon < \hat{\phi} - \phi < \varepsilon\right) = 1 - \alpha.$$

Then equivalently

$$P\left(\frac{-\varepsilon}{\sqrt{Var\left(\hat{\phi}\right)}} < \frac{\hat{\phi}-\phi}{\sqrt{Var\left(\hat{\phi}\right)}} < \frac{\varepsilon}{\sqrt{Var\left(\hat{\phi}\right)}}\right) = 1-\alpha$$

and assuming

$$\frac{\hat{\phi}-\phi}{\sqrt{Var\left(\hat{\phi}\right)}} \sim N(0,1)$$

then

$$P\left(\frac{-\varepsilon}{\sqrt{Var\left(\hat{\phi}\right)}} < Z < \frac{\varepsilon}{\sqrt{Var\left(\hat{\phi}\right)}}\right) = 1-\alpha$$

$$\Phi\left(\frac{-\varepsilon}{\sqrt{Var\left(\hat{\phi}\right)}}\right) = \frac{\alpha}{2} \tag{2.10}$$

where Φ denotes the cumulative standard normal distribution. Expression (2.9) leads to

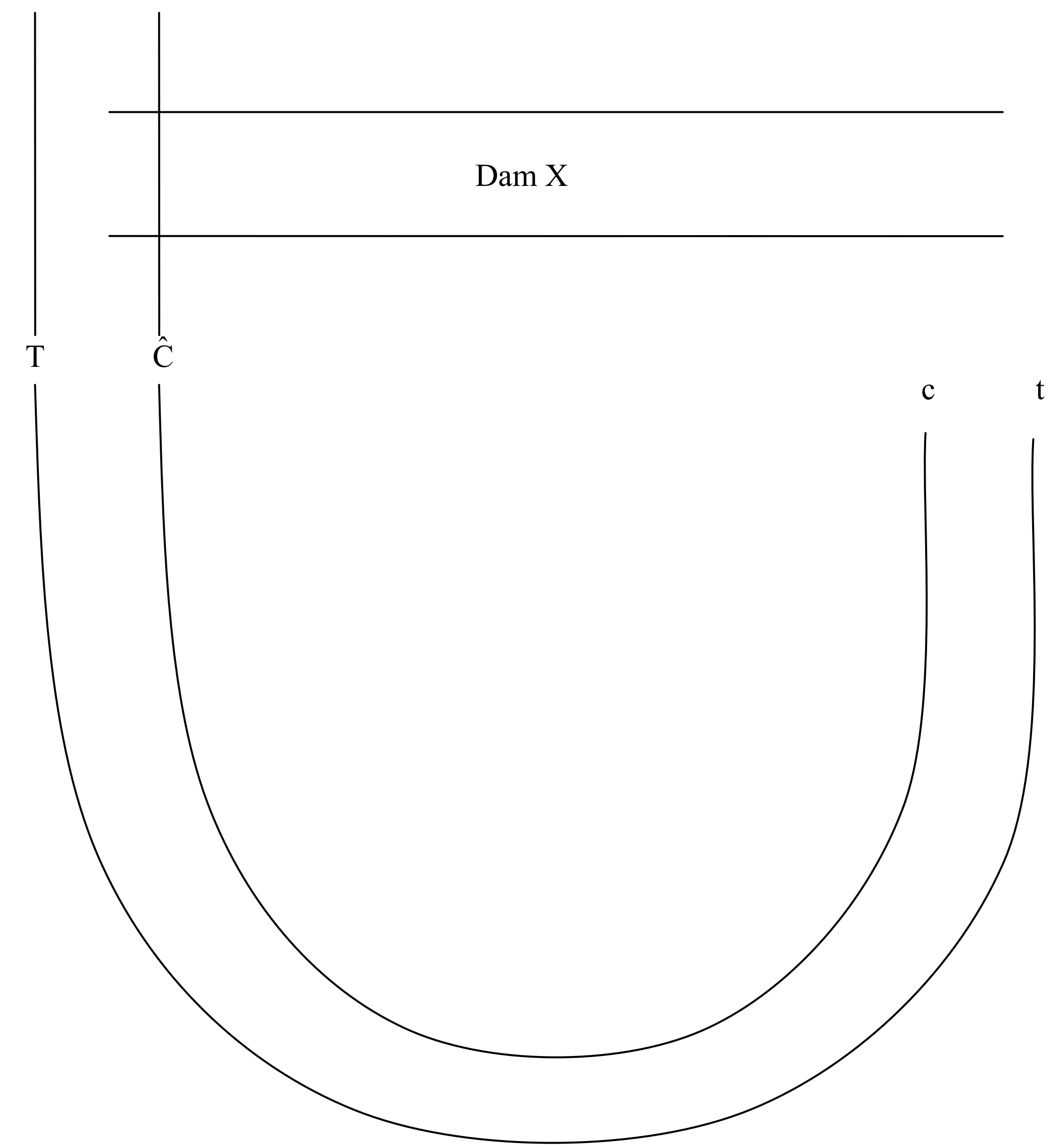
$$\varepsilon = Z_{1-\frac{\alpha}{2}}\sqrt{Var\left(\hat{\phi}\right)}. \tag{2.11}$$

2.5 Unknown Control Group Size, C

This scenario has arisen in the more recent transportation/control studies of the Fish Passage Center, Columbia Basin Fish and Wildlife Authority, known as the Comparative Survival Studies (CSS). In these studies, the inriver controls are composed of smolts estimated to have passed the release site undetected. The controls have consisted of juveniles not detected at any downstream bypass system or detected at most at one bypass system. In either case, the number of smolts in the control group must be estimated $(\hat{C})$ rather than known (C). The scenario where $\hat{C}$ is an estimated quantity is depicted in Figure 2.2.

Figure 2.2. Essential elements of transportation studies used to assess benefits to survival when the size of the untransported (control) group is estimated rather than known without error. The size of C can be estimated by a variety of release-recapture techniques.

Scenario 1: T known, C estimated by $\hat{C}$.



In this second scenario, the T/I ratio (2.1) becomes

$$\hat{R}_2 = \frac{\left(\frac{t}{T}\right)}{\left(\frac{c}{\hat{C}}\right)} = \frac{t\hat{C}}{cT} \tag{2.12}$$

and the variance can be written as follows (Appendix A)

$$\begin{aligned} Var\left(\hat{R}_2\right) &\doteq Var\left(\hat{R}\right) + R^2 \cdot CV\left(\hat{C}|C\right)^2 \\ &= R^2\left(\frac{1-R\theta}{TR\theta} + \frac{1-\theta}{C\theta}\right) + R^2 CV\left(\hat{C}|C\right)^2 \end{aligned} \tag{2.13}$$

and estimated by

$$\widehat{Var}\left(\hat{R}_2\right) \cong \hat{R}^2\left(\frac{1}{t} - \frac{1}{T} + \frac{1}{c} - \frac{1}{C}\right) + \hat{R}^2 \cdot CV\left(\hat{C}|C\right)^2. \tag{2.14}$$

The estimator of R when C must be estimated can be written as

$$\hat{R}_{2_{BC}} = \frac{t\hat{C}}{cT}\left(1 - \frac{1}{c} + \frac{1}{\hat{C}}\right). \tag{2.15}$$

The variance of $\hat{R}_{2_{BC}}$ can be approximated by

$$\begin{aligned} Var\left(\hat{R}_{2_{BC}}\right) \doteq & \frac{R^2(2-\theta-C\theta)^2(1-\theta) + (C\theta-1+\theta)^2 CR(1-R\theta)}{C^3\theta^3} \\ & + \frac{R^2(C\theta-1)^2}{C^2\theta^2}\left[CV\left(\hat{C}|C\right)\right]^2 \end{aligned} \tag{2.16}$$

with variance estimator

$$\begin{aligned} \widehat{Var}\left(\hat{R}_{2_{BC}}\right) = & \frac{\hat{R}^2}{c^3}\left(2 - \frac{c}{C} - c\right)^2\left(1 - \frac{c}{C}\right) + \frac{RC}{c^3}\left(c - 1 + \frac{c}{C}\right) + \left(1 - \frac{t}{T}\right) \\ & + \hat{R}^2\left(\frac{c-1}{c}\right)\left[CV\left(\hat{C}|C\right)\right]^2. \end{aligned} \tag{2.17}$$

The quantity $CV\left(\hat{C}|C\right)$ is the coefficient of variation of $\hat{C}$ [i.e., $SE\left(\hat{\theta}\right)/\theta$] given the true number of control fish is C. Equation (2.13) can be substituted into Equation (2.11) to explore absolute precision as a function of $CV\left(\hat{C}|C\right)$.

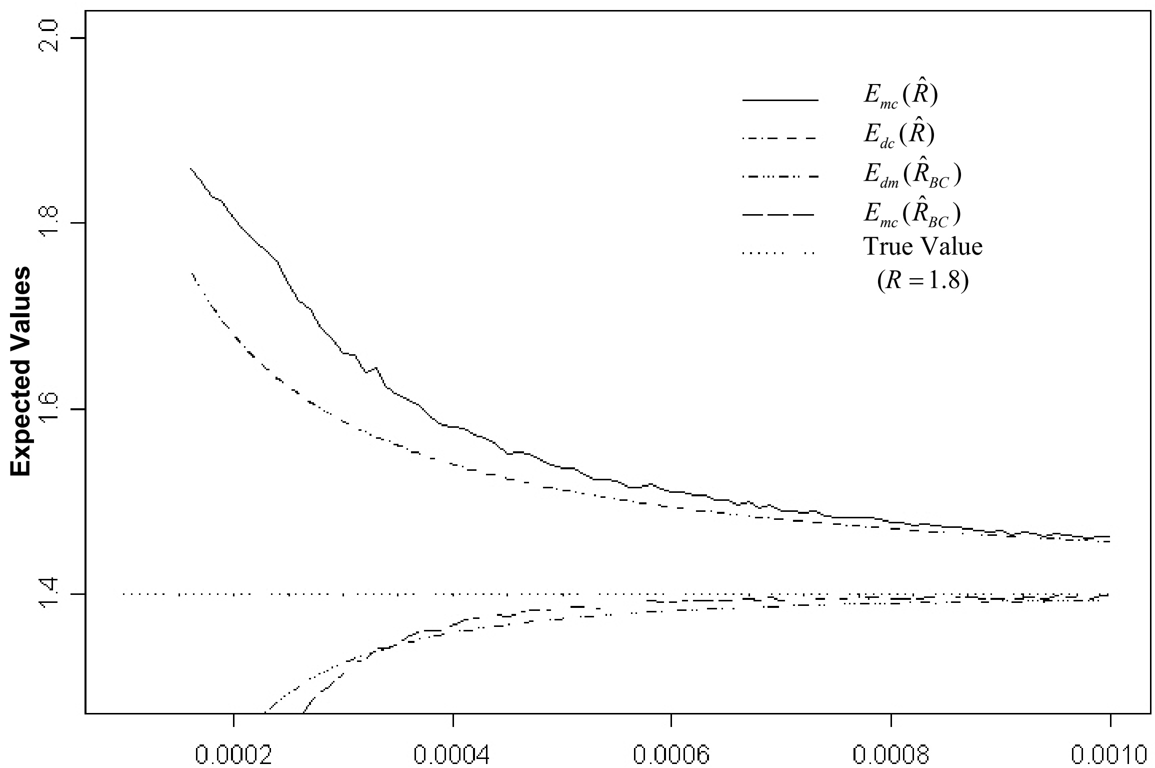
3.0 Monte Carlo Simulation Results

3.1 Bias of $\hat{R}$ and $\hat{R}_{BC}$

Section 2.2 demonstrated that $\hat{R}$ is a biased estimator of the true T/I ratio, R. In this section, we investigate the accuracy of $\hat{R}$ and of $\hat{R}_{BC}$ (Section 2.3) to estimate R as a function of control and transported recovery probabilities θ and $R\theta$, and release sizes T and C (in this investigation, they well be assumed equal). Because bias is defined as the difference between the expected value and the true value of an estimator , we can investigate bias by plotting the expected values of $\hat{R}$ and $\hat{R}_{BC}$, and comparing these values with the true value.

Figure 3.1 shows the delta-method expectation, $E_{dm}(\hat{R})$, (dotted-dashed line) plotted as a function of θ, the control group return rate, when the true T/I ratio, R, is 1.4 and release sizes $T = C$ = 25,000 fish. The range of values of θ were chosen from adult returns based on CWT release-recovery data on spring yearling chinook salmon between 1986-1988 (Townsend and Skalski 2000). The plot illustrates that as θ decreases in size, the bias of $\hat{R}$ becomes increasingly positive. For control return rates as high as 0.1%, there is still appreciable positive bias associated with the estimator $\hat{R}$ (Equation 2.1). In fact, the delta method approximation to the bias of $\hat{R}$ (Equation 2.5) actually underestimates the true magnitude of the bias associated with $\hat{R}$ (Figure 3.1).

Figure 3.1. Delta method (dm) and simulated (Monte Carlo, mc) values of the expected value of $\hat{R}$ and of $\hat{R}_{BC}$ when the true T/I ratio is 1.4, release sizes $T = C$ = 25,000, and θ varies over the range 0.00005-0.001.



The bias-corrected estimator $\hat{R}_{BC}$ is seen to have a negative bias based on the Monte Carlo simulation studies (Figure 3.1). However, the absolute magnitude of the bias is less for $\hat{R}_{BC}$ than $\hat{R}$ for all values of θ. Furthermore, the estimator $\hat{R}_{BC}$ becomes asymptotically unbiased for values of θ much smaller than that required for $\hat{R}$ (Figure 3.1).

Figures 3.2 and 3.3 investigate the effects of changing the true T/I ratio, R, and release sizes of fish in the transported and control groups T and C. Figure 3.2 shows delta method and Monte Carlo expected values of $\hat{R}$ and $\hat{R}_{BC}$, when fish group sizes were held at $T = C$ = 25,000 (Figure 3.1) but true T/I ratio, R = 1.9. Figure 3.1 and 3.2 are qualitatively very similar. Figure 3.3 shows delta-method and Monte Carlo expected values of $\hat{R}$ $\hat{R}_{BC}$, when true T/I ratio, R = 1.4, but release sizes were increased to $T = C = 100{,}000$. Numbers of fish released in a study make a big difference in the extent of bias (largest amount). In Figure 3.3, the bias of $\hat{R}_{BC}$ is nearly zero for values of θ larger than 0.0002. Bias in $\hat{R}$ is also smaller for the same values of θ, when group sizes are increased from 25,000 t0 100,000. However, there is still a positive bias when $\theta = 0.001$.

The irony of using $\hat{R}$ in analyzing T/I studies is that as release sizes (T, C) and/or recovery rates (θ) decrease, the estimated T/I ratio increases due to estimation bias. In other words, the poor the study design, the more likely the resulting estimate of R is prone to show a transport benefit. The bias-corrected estimator, on the other hand, is likely to produce an unbiased or slightly negative estimate of the benefit of transportation.

Figure 3.2. Delta method (dm) and simulated Monte Carlo (mc) values of the expected value of $\hat{R}$ and $\hat{R}_{BC}$ when the true T/I ratio R = 1.8, $T = C$ = 25,000, and θ varies over the range 0.00005 to 0.001.

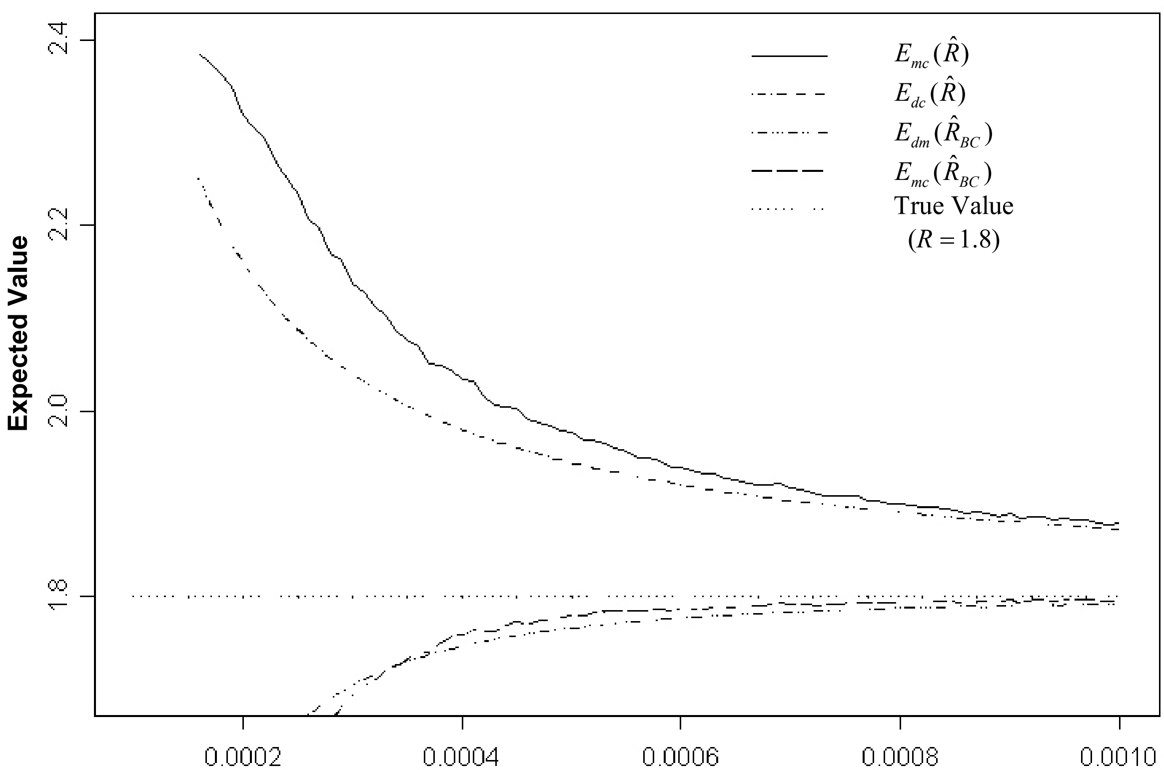
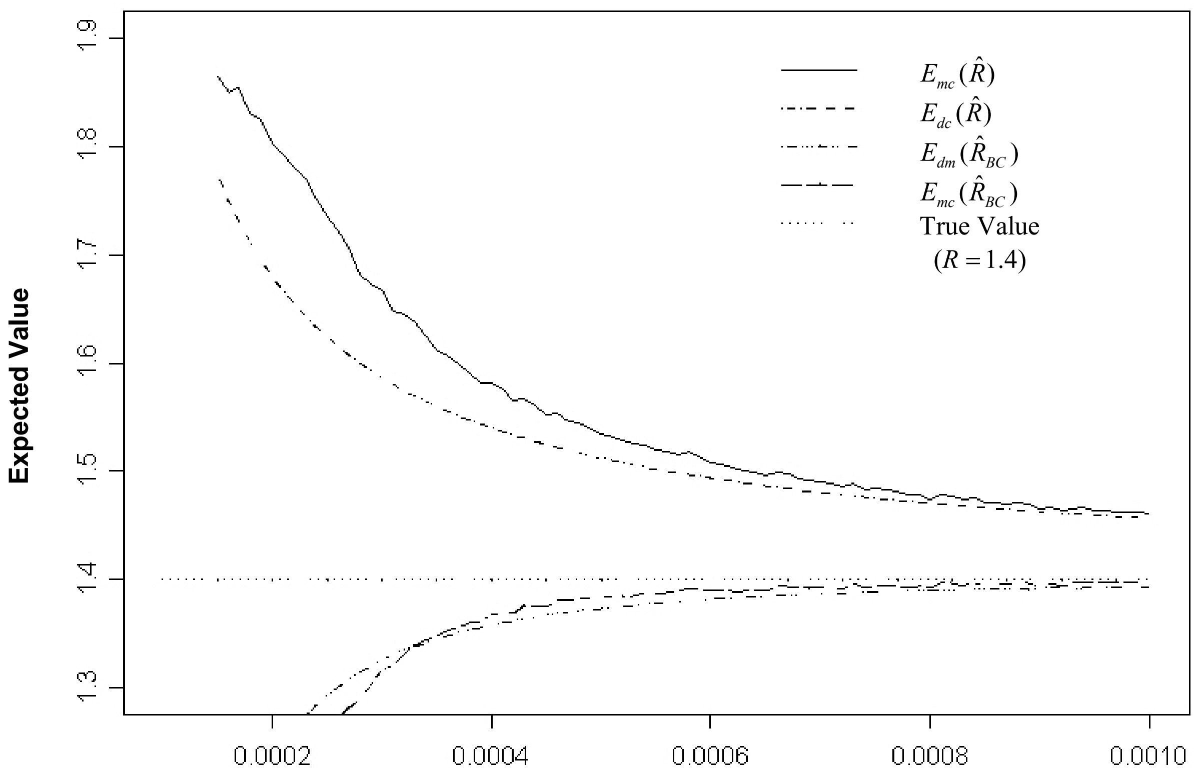


Figure 3.3. Delta method (dm) and simulated (Monte Carlo, mc) values of the expected value of $\hat{R}$ and $\hat{R}_{BC}$ when the true T/I ratio R = 1.4, $T = C$ = 100,000, and θ varies over the range 0.00005 to 0.001.



3.2 Sampling Variance of $\hat{R}$ and $\hat{R}_{BC}$

As in the previous section 3.1, the Monte Carlo simulation results provide characterizations of the true performance of the estimators and their variances. Figure 3.4 illustrates that the true variance of $\hat{R}$ [i.e., $Var_{mc}(\hat{R})$] is greater than the true variability in the bias-corrected estimator $\hat{R}_{BC}$ [i.e., $Var_{mc}(\hat{R}_{BC})$]. The magnitude of the variances for $\hat{R}$ and $\hat{R}_{BC}$ are asymptotic to similar values for large release sizes $(T = C)$ and recovery rates (θ).

The variance estimator for $\hat{R}$ [i.e., $Var_{dm}(\hat{R})$], however, is shown in Figure 3.4 to underestimate the true variance [i.e., $Var_{mc}(\hat{R})$] of the estimator. In contrast, the variance estimator for $\hat{R}_{BC}$ [i.e., $Var_{mc}(\hat{R}_{BC})$] has a tendency to overestimate the true variance associated with the estimator [i.e., $Var_{dm}(\hat{R}_{BC})$]. For a variance estimator to be valid, it should equal the true variance in expectation. If it is not unbiased, the variance estimator should at least be a conservative estimator, with values greater than the true value. Hence, the $\widehat{Var}(\hat{R})$ of Equation (2.3) is neither valid nor conservative. Figures 3.5 and 3.6 show the same variance relationships as Figure 3.4 except for different values of R and release sizes $(T = C)$.

Figure 3.4. Delta method (dm) and simulated (Monte Carlo, mc) values of the expected variance of $\hat{R}$ and $\hat{R}_{BC}$ when the true T/I ratio R = 1.4, $T = C$ = 25,000, and θ varies over the range 0.0005 to 0.002.

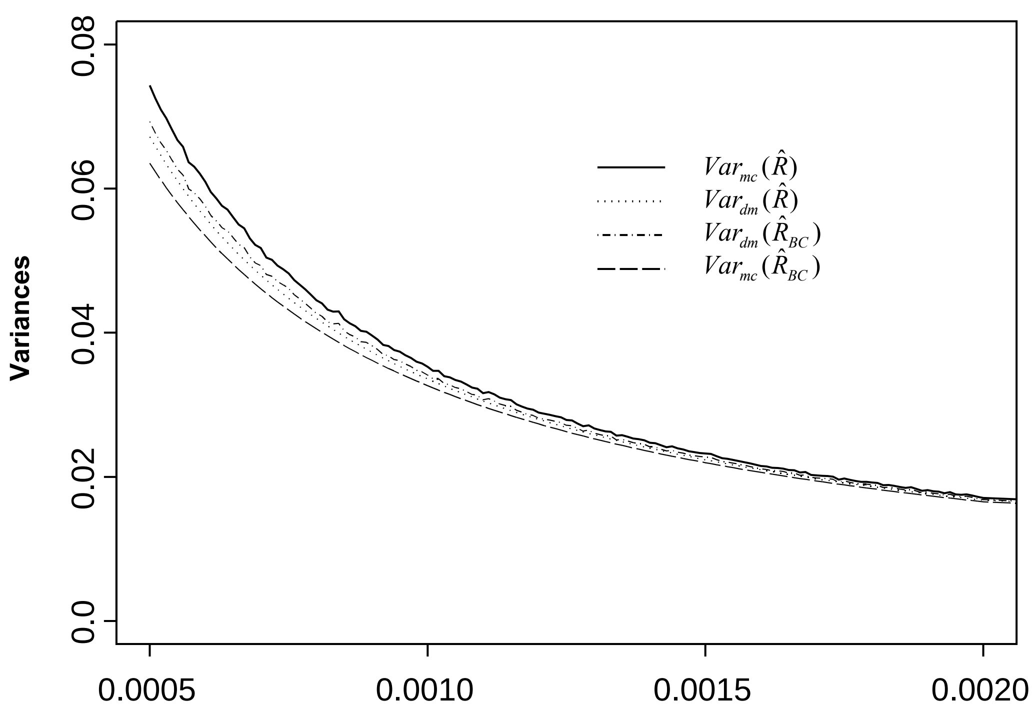


Figure 3.5. Delta method (dm) and simulated (Monte Carlo, mc) values of the expected value of $\hat{R}$ and $\hat{R}_{BC}$ when the true T/I ratio R = 1.8, $T = C$ = 25,000, and θ varies over the range 0.0005 to 0.002.

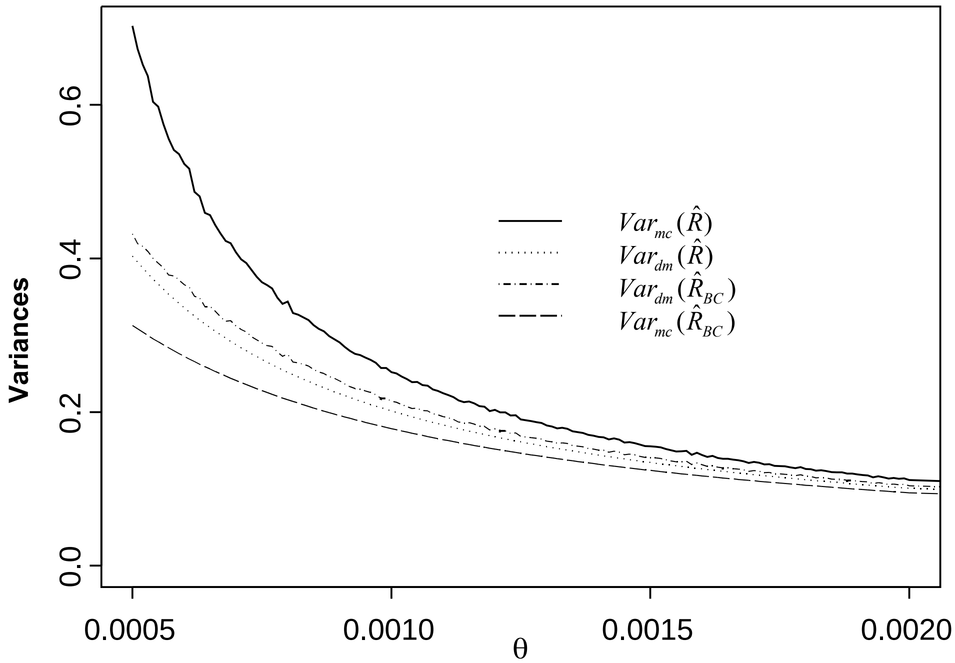
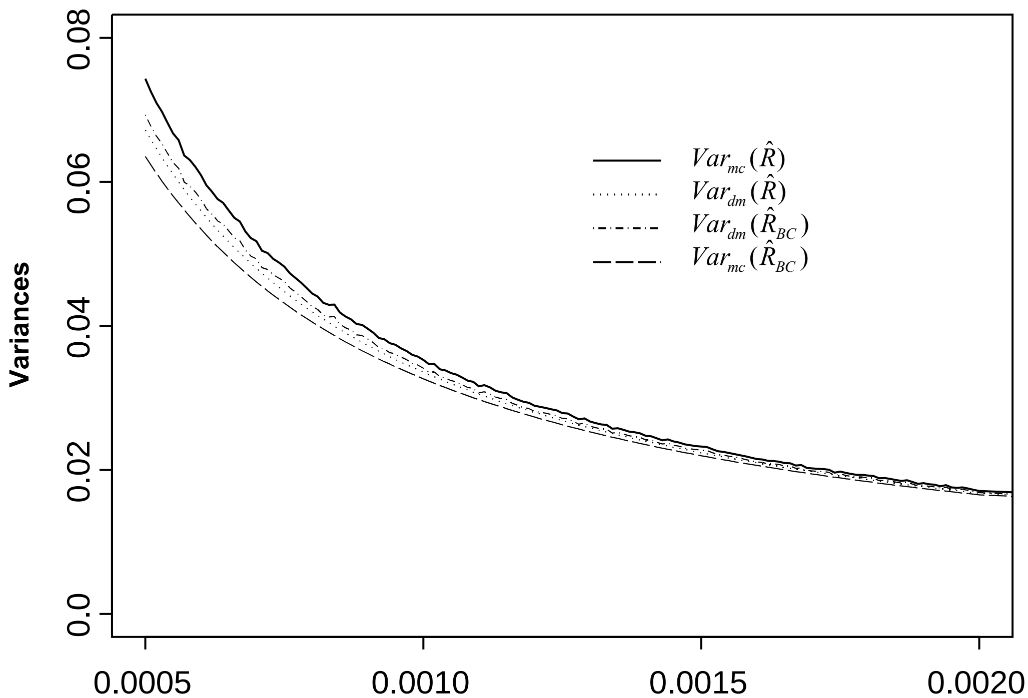


Figure 3.6. Delta method (dm) and simulated (Monte Carlo, mc) values of the expected value of $\hat{R}$ and $\hat{R}_{BC}$ when the true T/I ratio R = 1.4, = 100,000, and θ varies over the range 0.0005 to 0.002.



3.3 Coverage Probability of 95% Confidence Intervals for $\hat{R}$

A small variance or estimated standard error means greater precision for an estimation but precision by itself cannot account for how likely it is that a (1 - α) 100% confidence interval for R will contain the parameter value. Therefore, in this section, we will investigate how often a (1 - α) 100% confidence interval is likely to include the true value of R .

It is desirable that a (1 - α) 100% confidence interval for R contain the true value (1 - α) 100% of the time, but it is also desirable that it exclude values of R' not equal to R as often as possible. An estimator whose confidence interval coverage probability is maximum at the true value and drops precipitously for values smaller or larger than the true value of the parameter is both precise and has a greater likelihood of excluding false values.

To estimate the probability of coverage of a 95% confidence interval, a large number of such intervals are simulated using Monte Carlo methods, and then the percentage of those containing R or R' is computed. Figure 3.7a (solid line) displays the estimate probability of coverage of R = 1.4 and various other values of R' for the situation when θ is equal to 0.0005, $T = C$ = 25,000, and using the asymptotic normal approximation

$$\hat{R} \pm 1.96\sqrt{\widehat{Var}(\hat{R})}. \tag{3.1}$$

The dashed line in Figure 3.7a is the coverage probabilities using the asymptotic normal confidence interval for $\hat{R}_{BC}$ where

$$\hat{R}_{BC} \pm \sqrt{\widehat{Var}(\hat{R})}. \tag{3.2}$$

The log-linear form of estimator $\hat{R}$ suggests a log-transformation may better approximate a normal distribution. Hence, assuming the estimators of R are log-normally distributed, asymptotically log-normal confidence intervals of the following form were also computed where

$$\hat{R} \cdot e^{\pm 1.96\sqrt{\frac{Var_{dm}(\hat{R})}{\hat{R}_{BC}^2}}} \tag{3.3}$$

and

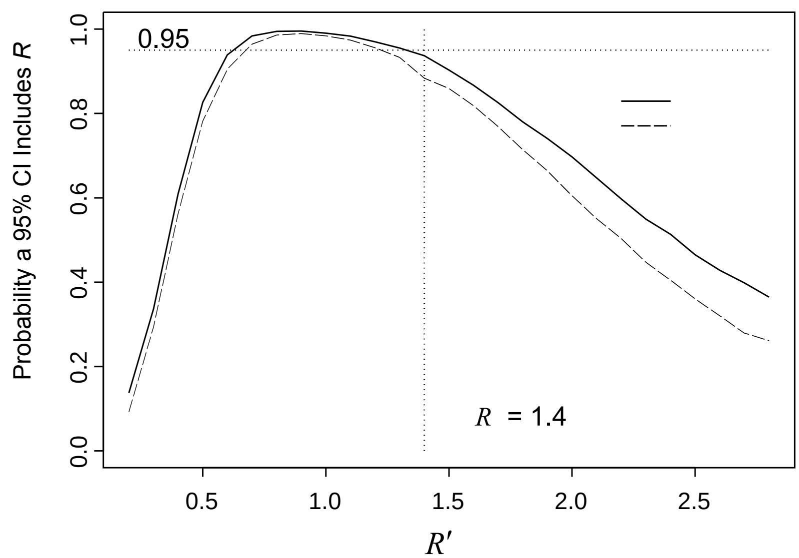
$$\hat{R}_{BC} \cdot e^{\pm 1.96\sqrt{\frac{Var_{dm}(\hat{R}_{BC})}{\hat{R}_{BC}^2}}} \tag{3.4}$$

for $\hat{R}$ and $\hat{R}_{BC}$, respectively. Asymptotic log-normal confidence intervals of the form (3.3) and (3.4) are plotted in Figure 3.7b. Figure 3.8 presents another series of coverage plots when $T = C$ = 20,000; R = 1.4; θ = 0.002.

Examination of the coverage plots in Figures 3.7 and 3.8 indicates nominal coverage probabilities are better attained with the log-transformed estimates of $\hat{R}$ and $\hat{R}_{BC}$, rather than the untransformed estimators. Coverage plots are comparable for both estimators $\hat{R}$ and $\hat{R}_{BC}$. We therefore recommend that confidence intervals be calculated using Equation (3.3) or (3.4).

Figure 3.7. Estimated coverage probabilities of 95% confidence intervals for R' when R = 1.4. Each estimated probability was computed from 100,000 simulations. Each simulation was generated as described in Section 3.1, using θ = 0.0005 and $T = C$ = 25,000. Top graph (a) was constructed from asymptotic normal confidence intervals of the form (3.1, solid line) and (3.2, dotted line). Bottom graph (b) was computed using asymptotic log-normal confidence intervals of the form (3.3, solid line) and (3.4, dashed line).

a.



b.

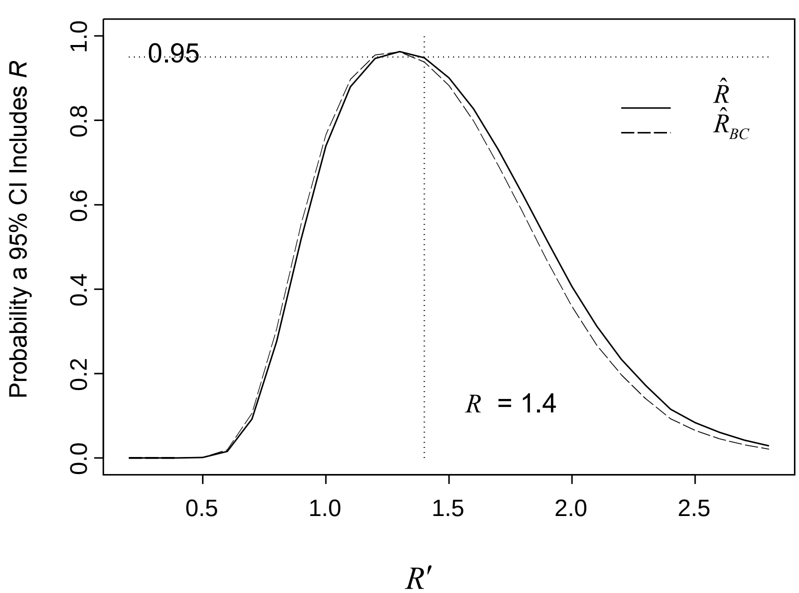
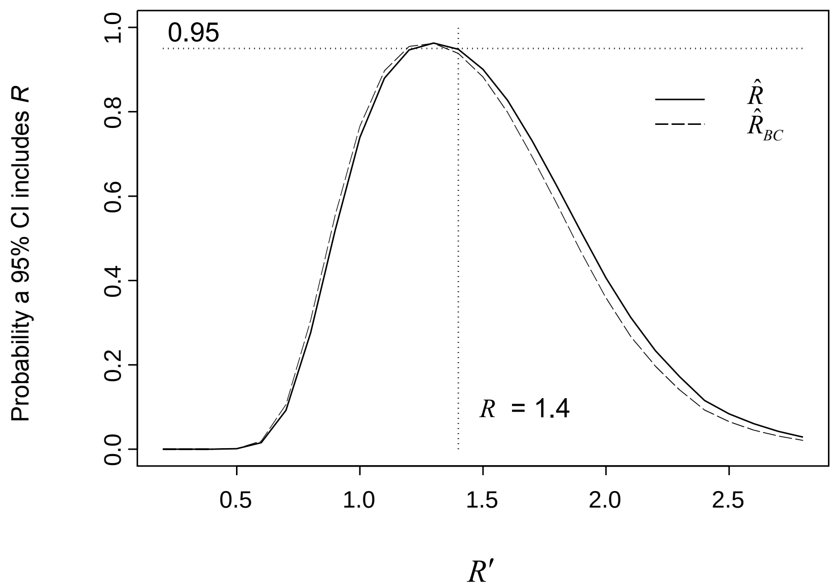
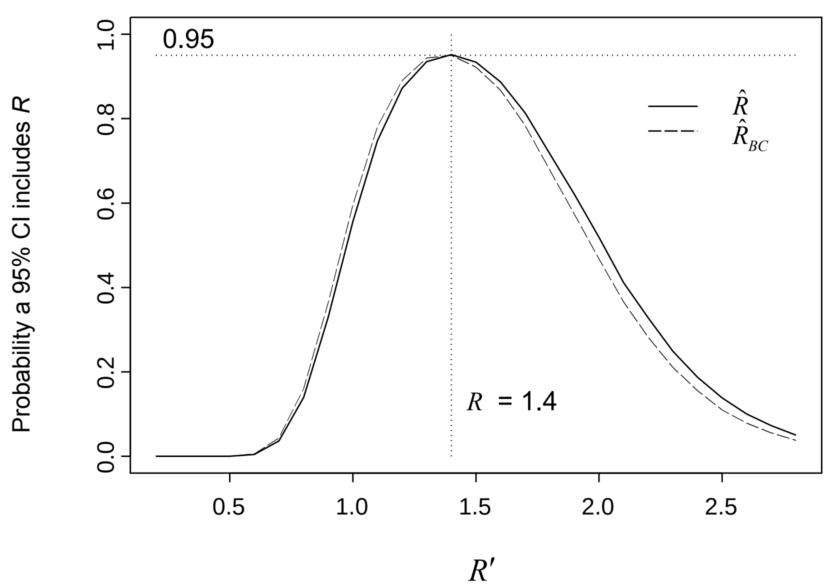


Figure 3.8. Estimated coverage probabilities of 95% confidence intervals for R' when R = 1.4. Each estimated probability was computed from 100,000 simulations. Each simulation was generated as described in Section 3.1, using θ = 0.002 and $T = C$ = 25,000. Top graph (a) was constructed from asymptotic normal confidence intervals of the form (3.1, dashed line) and (3.2, solid line). Bottom graph (b) was computed using asymptotic log-normal confidence intervals of the form (3.3, dashed line) and (3.4, solid line).

a.



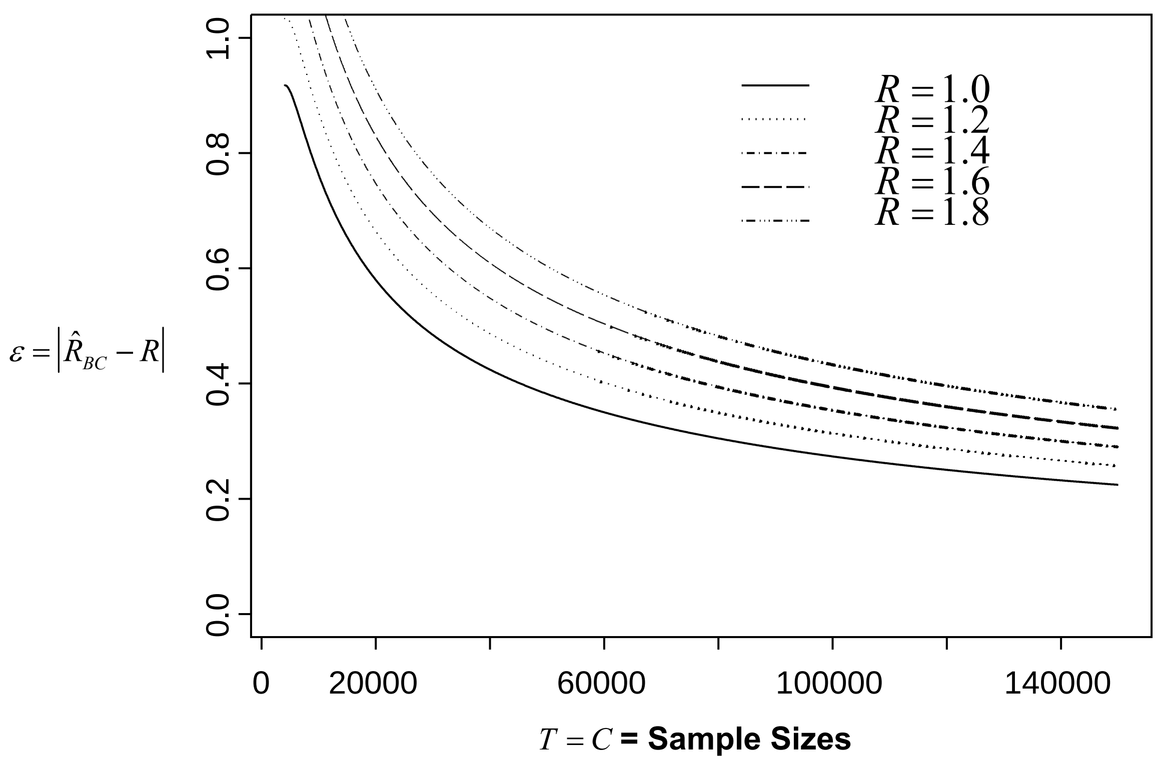
b.



3.4 Sampling Precision of $\hat{R}_{BC}$

In Sections 3.1 through 3.3, it was demonstrated that $\hat{R}_{BC}$ is a better estimator of R than $\hat{R}$. It is more accurate (has smaller bias, Section 3.1), more precise (smaller variance, Section 3.2), and when used in conjunction with a log-transformation, produces confidence intervals with the best coverage properties (Section 3.3). In this section, we explore the relationship between the precision of $\hat{R}_{BC}$ (Equation 2.7), release group size (i.e., T, C), and values of θ and R. (Throughout this section, 1 - α = 0.95.) Figure 3.9 shows how precision of $\hat{R}_{BC}$ varies with sample size when θ = 0.001 and R = 1.0, 1.2, 1.4, 1.6, and 1.8.

Figure 3.9. Absolute precision of $\hat{R}_{BC}$ (Equation 2.7) as a function of release size ($T = C$) when θ = 0.001 and R = 1.0, 1.2, 1.4, 1.6, and 1.8.



The larger R is, the larger the sample sizes must be to obtain a specified level of precision. For example, a sample size of 50,600 is required to ensure that, with 95% probability, $\hat{R}_{BC}$ will be within 0.6 of R = 1.8. To ensure it will be within 0.6 of R = 1.0 would required 18,400 fish. Rather large sample sizes are required for sufficient precision to distinguish R = 1.4 from R = 1.0 ($T = C$ = 77,700). Figure 3.10 illustrates the effect of increasing θ from 0.001 to 0.01. For θ = 0.01, a sample size of 7,600 is required to ensure with 95% probability that $\hat{R}_{BC}$ will be within $\pm$ 0.04 of R = 1.4.

Figure 3.10. Absolute precision of $\hat{R}_{BC}$ as a function of release size $(T = C)$ when θ = 0.01 and R = 1.0, 1.2, 1.4, 1.6, and 1.8.

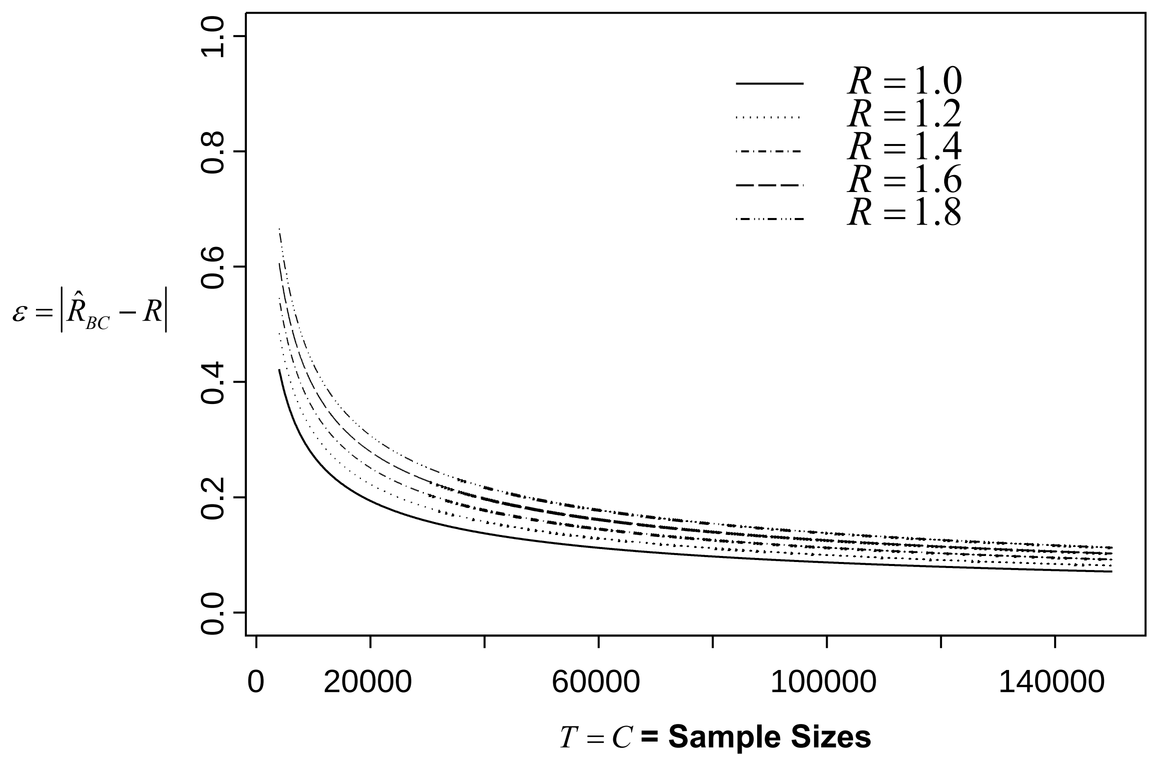


Table 3.1 presents sample sizes required to be within ε = 0.4 of true R with probability 100 $(1-\alpha)$ when true R = 1.4 for given values of θ. Figure 3.11 illustrates the relationship between the precision of $\hat{R}_{BC}$ and sample size when R is held at 1.4 and θ = 0.0005, 0.001, 0.002, 0.005, 0.01, or 0.03. (The dot-dashed curve in 3.10 gives the same information as the dot-dot-dot-dashed curve in Figure 3.12.)

Table 3.1. Sample sizes $(N = T = C)$ required to achieve an absolute precision (Section 2.4) of ε = 0.4 when R = 1.4 for the given adult return rate, θ, and $1-\alpha$ = 0.80, 0.95.

θ	α	Sample sizes $(N = T = C)$ required to achieve precision, ε = 0.4, when R = 1.4
0.01	0.05	7,600
	0.20	<4,000
0.001	0.05	77,700
	0.20	31,400
0.0005	0.05	155,500
	0.20	63,300
0.0001	0.05	778,200
	0.20	315,100

Figure 3.11. Sampling precision for $\hat{R}_{BC}$ (Equation 2.7) as a function of release size $(T = C)$ when R = 1.4 and θ = 0.0005, 0.001, 0.002, 0.005, 0.01, and 0.03.

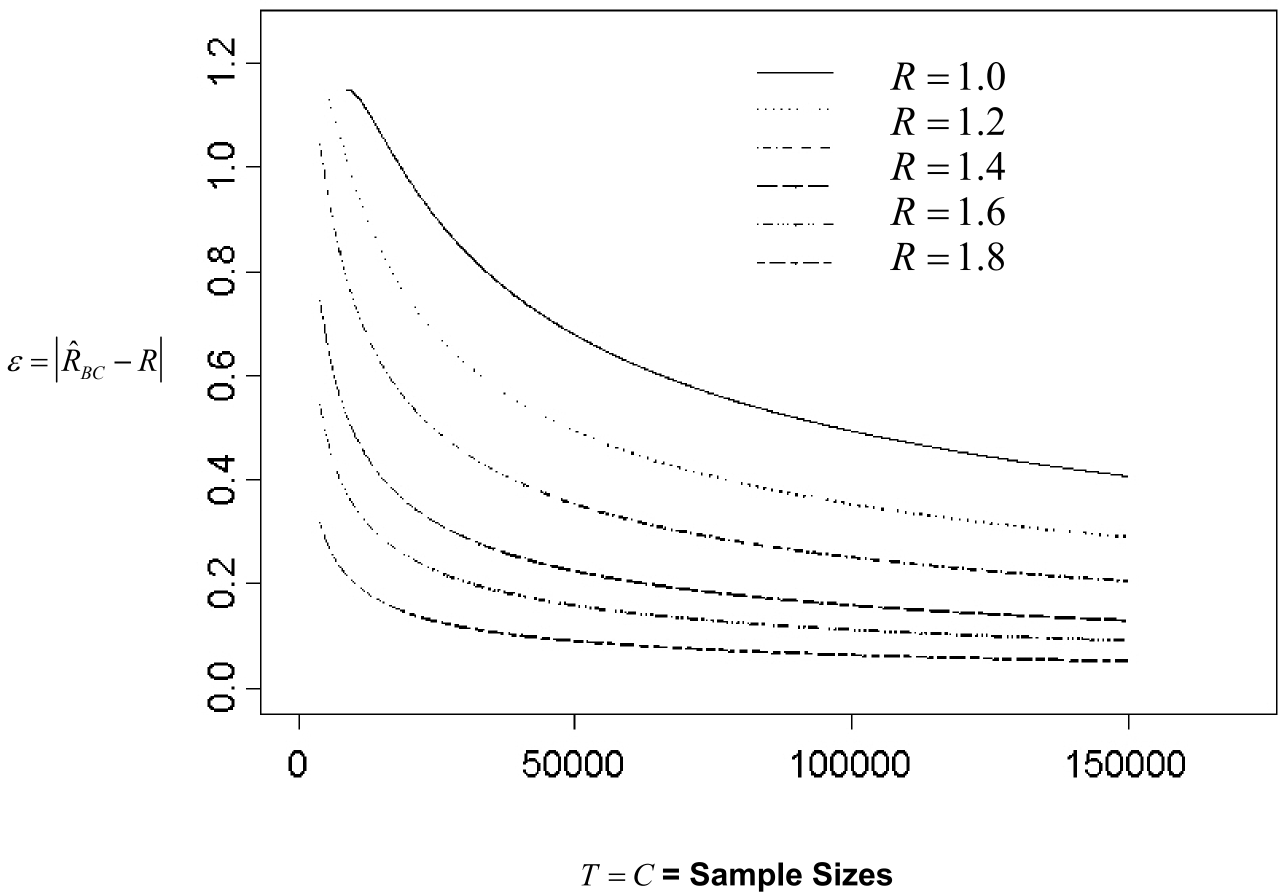


Figure 3.12 explores the relationship- between precision of $\hat{R}_{BC}$ and sample size when Scenario 2 exists; that is, when the control group size, C, is an unknown random variable, $\hat{C}$. Equation (2.8) is substituted into Equation (2.7). When the coefficient of variation (CV) of $\hat{C}$ is equal to 0, the variance of $\hat{C}$ is 0, and the curve (solid line) is the same as the dot-dashed plot in Figure 3.10. Increasing the CV to 0.1, i.e., the standard deviation of $\hat{C}$ is one-tenth of the mean of $\hat{C}$, produces a great deal more uncertainty about $\hat{R}_{BC}$. And if the CV ($\hat{C}$) = 0.5, the precision is absurdly small for sample sizes as great as 140,000. Table 3.2 shows sample sizes required for precision, ε, when true R = 1.4 for selected θ and α, and in the case where C must be estimated.

Figure 3.12. Absolute precision of $\hat{R}_{BC}$ as a function of release size $(T = C)$ when R = 1.4, θ = 0.01, and coefficient of variation (CV) of $\hat{C}$ = 0.0, 0.1, 0.2, 0.3, 0.4, and 0.5.

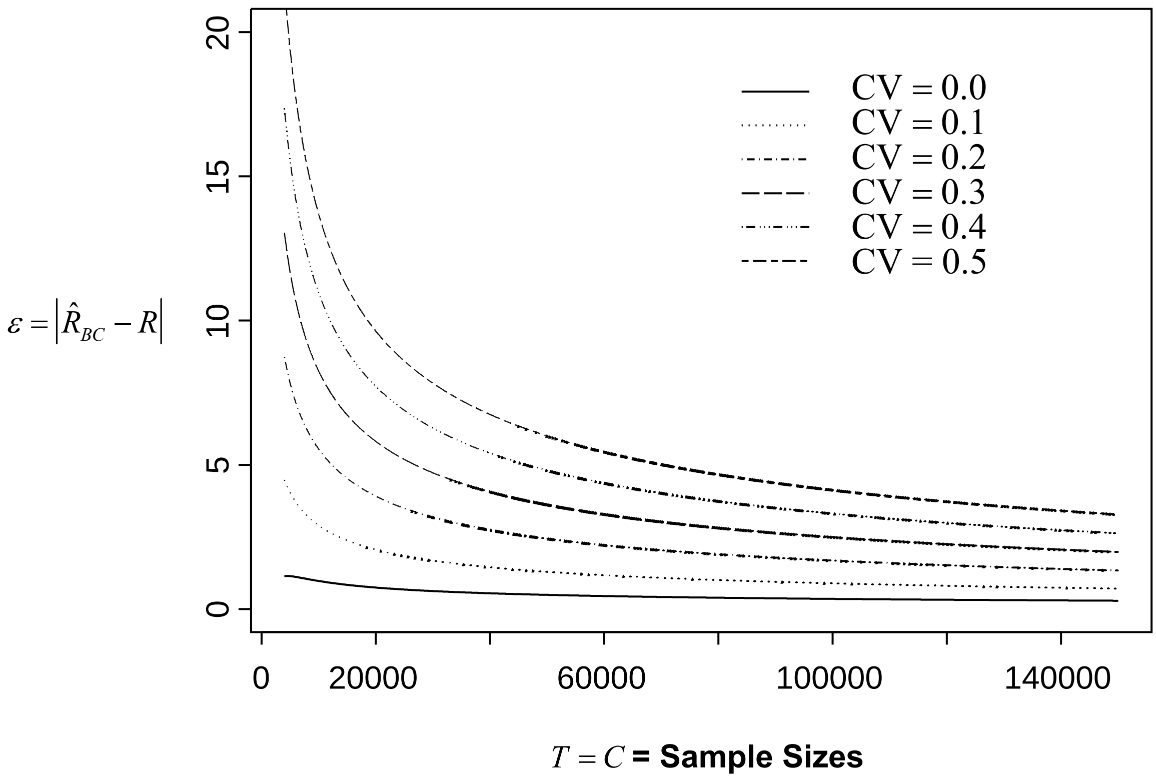


Table 3.2. Sample sizes (to nearest 100), $N = T = C$, required to achieve an absolute precision (Section 2.4) of ε = 0.4 when R = 1.4 for the given adult return rate, θ, and $1-\alpha$ = 0.80 or 0.95 in the case where C must be estimated.

θ	α	CV of $\hat{C}$	Sample sizes $(N = T = C)$ required to achieve precision, ε = 0.4, when R = 1.4
0.04	0.05	0.1	325,400
	0.20	0.1	374,300
	0.05	0.5	927,900
	0.05	0.5	839,800
	0.20	0.1	195,700
	0.20	0.1	436,600
	0.05	0.01	850,800
	0.05	0.5	983,000

3.5 Example

Table 3.3 presents juvenile release and adult return data for groups of transported and control fish PIT-tagged in 1995 and 1996 and released inriver or barged from Lower Granite Dam. Data is from the second-tier database, Data Access in Real Time (DART), managed by the University of Washington and from Marsh et al. (1997). The column labeled T contains numbers of PIT-tagged spring/summer chinook salmon smolts (hatchery, wild, and combined) barged from Lower Granite Dam. The column labeled C contains numbers of PIT-tagged fish (hatchery, wild, and combined) released into the tailrace of Lower Granite Dam. Columns labeled t and c are adult returns from the transported and inriver groups, respectively. The last two columns present estimates of the T/I ratio using estimators $\hat{R} = \frac{tc}{cT}$ and $\hat{R}_{BC} = \frac{tC}{cT}\left(1 - \frac{1}{c} + \frac{1}{C}\right)$.

Table 3.3. Release and adult return numbers of wild, hatchery, and combined spring/summer yearling chinook salmon PIT-tagged and released or barged from Lower Granite Dam in 1995 and 1996, and the original transported-to-inriver (T/I) ratio estimates, $\hat{R}$, and bias-corrected estimates, $\hat{R}_{BC} \cdot T$ and C are the numbers of PIT-tagged transported and control smolts released, respectively, and t and c are the adult returns from the transported and inriver groups, respectively.

Year	Group	Transported Group		Inriver Group		$\hat{R}$	$\hat{R}_{BC}$
		Releases, T	Returns, t	Releases, C	Returns, c		
1995	Hatchery	83,149	457	105,875	328	1.7741	1.7687
	Wild	24,075	91	31,733	64	1.8742	1.8449
	Combined	107,224	548	137,608	392	1.7941	1.7895
1996	Hatchery	37,190	51	53,976	53	1.3966	1.3703
	Wild	8,791	10	14,078	7	2.2877	1.9611
	Combined	45,981	61	68,054	60	1.5047	1.4797

Data sets in which the value of c is very small relative to C show the largest differences between estimated values $\hat{R}_{BC}$ and $\hat{R}$. Of the 14,078 wild spring/summer chinook salmon PIT-tagged and released in 1996 to migrate inriver, only 7 adult returns were recorded. The difference between $\hat{R}_{BC}$ and $\hat{R}$ was much larger for this data set than the others. The adult return rate for the control group of this data set was θ = 0.0005 (Table 3.4). The 95% asymptotic lognormal confidence interval for this data was also much wider than the other datasets (Table 3.4).

Table 3.4. Estimates computed from transported and inriver releases and returns in Table 3.3 (see text for explanation).

Year	Group	$\hat{\theta} = \frac{c}{C}$	95% Asymptotic Lognormal Confidence Interval Using $\hat{R}_{BC}$	95% Asymptotic Lognormal Confidence Interval Using $\hat{R}$
1995	Hatchery	0.0031	(1.54, 2.04)	(1.52, 2.03)
	Wild	0.0020	(1.34, 2.53)	(1.28, 2.47)
	Combined	0.0028	(1.57, 2.04)	(1.56, 2.03)
1996	Hatchery	0.0010	(0.93, 2.01)	(0.86, 1.93)
	Wild	0.0005	(0.80, 4.80)	(0.08, 4.50)
	Combined	0.0009	(1.04, 2.11)	(0.97, 2.04)

Table 3.4 contains estimates of θ for the data in Table 3.3, and 95% asymptotic lognormal confidence intervals using $\hat{R}_{BC}$ and 95% asymptotic normal confidence intervals using $\hat{R}$, intervals of

form (3.4) and (3.1) (Section 3.4), respectively. In one instance, the two methods produced results with a substantive difference. The combined wild and hatchery releases from 1996 recorded a 95% asymptotic lognormal CI using $\hat{R}_{BC}$ of (1.04, 2.11) compared to the 95% asymptotic CI using $\hat{R}$ of (0.97, 2.04). Although the difference is small, the former CI excludes 1.0, indicating a statistically significant transportation benefit while the standard analysis does not.

4.0 Discussion and Summary

This research found a bias correction to the traditional estimator of the effects of transportation. The traditional estimator was found to be positively biased over a wide range of release sizes and recovery rates. The variance estimator for $\hat{R}$ was also found to be negatively biased. The new bias-corrected estimator has a slight negative bias only under extremely small release sizes or recovery rates. Its variance estimator is both valid and conservative. CI estimators based on the assumption of lognormally distributed $\hat{R}$ estimates provided both [?] interval coverage than asymptotic normal interval estimators. These results on improved estimators, variance estimators, and CIs should improve the statistical reliability of transportation benefit analyses.

Using the best statistical methods developed in this research, sample size requirements for distinguishing R = 1.4 from R = 1.0 with 95% confidence are to the nearest hundred, 7,600; 77,700; and 155,500 when the control group adult return rates is 0.01, 0.001, and 0.0005, respectively. If the control group size is unknown and must be estimated, sample size requirements are higher. If the control group release size is estimated with a CV of 10%, then the sample size requirements to achieve the same precision (i.e., R = 1.4, ε = 0.4, $1-\alpha$ = 0.95) will be 325,400; 374,300; and 850,800 for control group adult return rates of 0.01, 0.001, 0.0001, respectively.

In an example of formulas derived in this research to PIT-tag data from the 1995 and 1996 transportation experiments, a large difference was found between the original T/I estimators (Eq. 2.1) and the bias-corrected estimator developed here (Eq. 2.5) for a dataset in which c is very small relative to C, i.e., when $\hat{\theta}$ is approximately 0.0005. The best CI formulation developed in this research, the asymptotic lognormal CI constructed using the bias-corrected estimate was applied to the example data, as was the formula for an asymptotic normal CI constructed using the traditional T/I ratio estimator (Eq. 2.1). In one example dataset, this best CI did not include R = 1.0 in the interval, while the asymptotic normal CI did.

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Appendix A: Derivations of Some Chapter 2 Results

The Delta Method

The function $f(x) = f(x_1, x_2, \ldots, x_n)$ can be written as a Taylor series expansion of f about $E(x_i) = u_i; i = 1, \ldots, n$ as follows:

$$f(x) = \sum_{k=1}^{\infty} \frac{f^{(k)}(x)(x-\mu)^k}{k!}.$$

The first three terms provide a reasonable approximation:

$$f(x) \doteq f(\mu) + \sum_{i=1}^{n}(x_i - \mu_i)\frac{df}{dx_i} + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(x_i - \mu_i)(x_i - \mu_j)}{2!}\frac{\partial^2 f}{\partial x_i \partial x_j},$$

where all derivatives are evaluated at $x_i = \mu_i; i = 1, \ldots, n$. Then $E(f(x))$ can be approximated by taking the expectation of the first-term expansion, $f(\mu)$, or the third-term expansion above. Because the second term on the right-hand side is equal to $E(x-\mu) = E(x) - E(\mu) = \mu - \mu = 0$, the expectation of the third-term expansion is

$$E(f(x)) = f(\mu) + E\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(x_i - \mu_i)(x_i - \mu_j)}{2!}\frac{\partial^2 f}{\partial x_i \partial x_j}\right),$$

and if x_i and x_j are independent for all i, j the above equation reduces to

$$f(\mu) + \sum_{i=1}^{n}\frac{f''(\mu_i)E(x_i - \mu_i)^2}{2} = f(\mu) + \sum_{i=1}^{n}\frac{f''(\mu_i)\cdot Var(x_i)}{2}. \quad \text{(A1)}$$

The variance of $f(x)$ is typically approximated by taking the variance of the second-term expansion, and in the case of independent variables x_i and x_j is written

$$Var\left(f(\mu) + \sum_{i}^{n}\frac{f'(\mu_i)(x_i - \mu_i)^1}{1!}\right) = \sum_{i=1}^{n}\left[f'(\mu_i)\right]^2 Var(x_i). \quad \text{(A2)}$$

Variance of $\hat{R}_{BC}$

The derivation of Equation (2.6) is as follows. From Equation (2.5), we have

$$Var\left(\hat{R}_{BC}\right) = Var\left[\frac{tC}{cT}\left(1-\frac{1}{c}+\frac{1}{C}\right)\right]. \tag{A3}$$

Then

$$Var\left(\hat{R}_{BC}\right) = \left(\frac{C}{T}\right)^2 Var\left[t\left(\frac{1}{c}-\frac{1}{c^2}+\frac{1}{Cc}\right)\right]$$

and we need to find the variance of the product of t and $\frac{1}{c}-\frac{1}{c^2}+\frac{1}{Cc}$. The general formula for the variance of two independent factors is

$$Var(xy) = \left(E(x)\right)^2 Var(y) + E(y)^2 Var(x) + Var(x)\cdot Var(y), \tag{A4}$$

and substituting t for x and $\frac{1}{c}-\frac{1}{c^2}+\frac{1}{Cc}$ for y gives

$$\begin{aligned} Var\left(\hat{R}_{BC}\right) = \left(\frac{C}{T}\right)^2 \Bigg\{ & \left[E(t)\right]^2 Var\left(\frac{1}{c}-\frac{1}{c^2}+\frac{1}{Cc}\right) + \left[E\left(\frac{1}{c}-\frac{1}{c^2}+\frac{1}{Cc}\right)\right]^2 \\ & \cdot Var(t) + Var(t)\cdot Var\left(\frac{1}{c}-\frac{1}{c^2}+\frac{1}{Cc}\right)\Bigg\}. \end{aligned} \tag{A5}$$

From assumptions 1-4 , Section (2.1) we have $E(t) = TR\theta$, $Var(t) = TR\theta(1-R\theta)$, $E(c) = C\theta$, and $Var(c) = C\theta(1-\theta)$. Substituting the appropriate terms in (A5) gives

$$\begin{aligned} Var\left(\hat{R}_{BC}\right) = \left(\frac{C}{T}\right)^2 \Bigg\{ & (TR\theta)^2 Var\left(\frac{1}{c}-\frac{1}{c^2}+\frac{1}{Cc}\right) + \left[E\left(\frac{1}{c}-\frac{1}{c^2}+\frac{1}{Cc}\right)\right]^2 \\ & \cdot TR\theta(1-R\theta) + TR\theta(1-R\theta)\cdot Var\left(\frac{1}{c}-\frac{1}{c^2}+\frac{1}{Cc}\right)\Bigg\}. \end{aligned} \tag{A6}$$

The terms $Var\left(\frac{1}{c}-\frac{1}{c^2}+\frac{1}{Cc}\right)$ and $E\left(\frac{1}{c}-\frac{1}{c^2}+\frac{1}{Cc}\right)$ can be derived using the delta method, as follows.

The variance of the second-term Taylor series approximation of $\frac{1}{c}-\frac{1}{c^2}+\frac{1}{Cc}$ around $E(c)$ is

$$Var(c)\left(-\frac{1}{c^2}+\frac{2}{c^3}-\frac{1}{Cc^2}\right)^2\Bigg|_{E(c)}=C\theta(1-\theta)\left(-\frac{1}{C^2\theta^2}+\frac{2}{C^3\theta^3}-\frac{1}{C^3\theta^2}\right)^2. \tag{A7}$$

The first-term Taylor approximation to $E\left(\frac{1}{c}-\frac{1}{c^2}+\frac{1}{Cc}\right)\doteq\left(\frac{1}{E(c)}\right)-\left(\frac{1}{E(c^2)}\right)+\left(\frac{1}{E(Cc)}\right)$ is

$$\frac{1}{C\theta}-\frac{1}{C^2\theta^2}+\frac{1}{C^2\theta}. \tag{A8}$$

The variance of $\hat{R}_{BC}$ can be approximated by the expression

$$\begin{aligned}Var\left(\hat{R}_{BC}\right)=\left(\frac{C}{T}\right)^2\Bigg\{&(TR\theta)^2 C\theta(1-\theta)\left(-\frac{1}{C^2\theta^2}+\frac{2}{C^3\theta^3}+\frac{1}{C^3\theta^2}\right)^2+\left(\frac{1}{C\theta}-\frac{1}{C^2\theta^2}+\frac{1}{C^2\theta}\right)^2\\ &\cdot TR\theta(1-R\theta)+TR\theta(1-R\theta)C\theta(1-\theta)\left(-\frac{1}{C^2\theta^2}+\frac{2}{C^3\theta^3}-\frac{1}{C^3\theta^2}\right)^2\Bigg\}.\end{aligned} \tag{A9}$$

and estimated by the formula

$$\begin{aligned}\widehat{Var}\left(\hat{R}_{BC}\right)=\left(\frac{C}{T}\right)^2\Bigg\{&t^2\left(\frac{c(C-c)}{C}\right)\left(-\frac{1}{c^2}-\frac{1}{c^3}+\frac{1}{Cc^2}\right)^2+\left(\frac{t(T-t)}{T}\right)\left(-\frac{1}{c}-\frac{1}{c^2}+\frac{1}{Cc}\right)^2\\ &+\left(\frac{t(T-t)}{T}\right)\left(\frac{c(C-c)}{C}\right)\left(-\frac{1}{c^2}-\frac{1}{c^3}+\frac{1}{Cc^2}\right)^2\Bigg\}.\end{aligned} \tag{A10}$$

Expected Value of $\hat{R}$

The first-term Taylor approximation is

$$
\begin{aligned}
E\left(\hat{R}\right) &= E\left(\frac{Ct}{Tc}\right) \\
&\doteq \frac{E(Ct)}{E(Tc)} \\
&= \frac{CTR\theta}{TC\theta} \\
&= R.
\end{aligned}
$$

Variance of $\hat{R}$

$$
\begin{aligned}
Var\left(\hat{R}\right) &= Var\left(\frac{Ct}{Tc}\right) \\
&= \left(\frac{C}{T}\right)^2 Var\left(\frac{T}{C}\right).
\end{aligned}
$$

Using the delta method approximation

$$
\begin{aligned}
Var\left(\hat{R}\right) &= \left(\frac{C}{T}\right)^2 \left[Var(t)\left(\frac{1}{c}\right)^2_{|E} + Var(c)\left(-\frac{t}{C^2}\right)^2_{|E} \right] \\
&= \left(\frac{C}{T}\right)^2 \left[\frac{TR\theta(1-R\theta)}{C^2\theta^2} + \frac{C\theta(1-\theta)T^2R^2\theta^2}{C^4\theta^4} \right] \\
&= R^2 \left[\frac{(1-R\theta)}{TR\theta} + \frac{(1-\theta)}{C\theta} \right].
\end{aligned}
$$

Variance of $\hat{R}_2 = \left(\frac{t\hat{C}}{cT}\right)$

This derivation relies on the identity,

$$
Var[x] = E\left[Var\left(x|y\right)\right] + Var\left[E\left(x|y\right)\right].
$$

Conditioning on C gives

$$Var\left(\frac{t\hat{C}}{cT}\right) = E\left[Var\left(\frac{t\hat{C}}{cT}\middle|C\right)\right] + Var\left[E\left(\frac{t\hat{C}}{cT}\middle|C\right)\right]. \tag{A11}$$

$Var\left(\frac{t\hat{C}}{cT}\right)$ is simply $\left(\frac{t}{cT}\right)^2 Var\left(\hat{C}\middle|C\right)$, and $E\left(\frac{t\hat{C}}{cT}\middle|C\right)$ is $\frac{tC}{cT}$. Substituting these quantities into (A11) gives

$$Var\left(\frac{t\hat{C}}{cT}\right) = E\left[\left(\frac{t}{cT}\right)^2 Var\left(\hat{C}\middle|C\right)\right] + Var\left(\frac{tC}{cT}\right) \cdot E\left[\left(\frac{t}{cT}\right)^2 Var\left(\hat{C}\middle|C\right)\right] + Var\left(\frac{tC}{cT}\right).$$

The first-term Taylor series approximation to $\left[E\left(\frac{t}{cT}\right)^2\right]$ is $\left(\frac{TR\theta}{C\theta T}\right)^2 = \frac{R^2}{C^2}$, and substituting this and $Var\left(\frac{tC}{cT}\right) = R^2\left[\frac{1-R\theta}{TR\theta} + \frac{1-\theta}{C\theta}\right]$ (Chapter 2) into the above gives

$$\begin{aligned}\left(\frac{R^2}{C^2}\right)Var\left(\hat{C}\middle|C\right) + R^2\left[\frac{1-R\theta}{TR\theta} + \frac{1-\theta}{C\theta}\right] &= R^2\left[\frac{1-R\theta}{TR\theta} + \frac{1-\theta}{C\theta}\right] + R^2\left(\frac{Var\left(\hat{C}\middle|C\right)}{C^2}\right) \\ &= R^2\left[\frac{1-R\theta}{TR\theta} + \frac{1-\theta}{C\theta}\right] + R^2\left[CV\left(\hat{C}\middle|C\right)\right]^2.\end{aligned} \tag{A12}$$

Variance of $\hat{R}_{2BC} = \left[\frac{t\hat{C}}{cT}\left(1 - \frac{1}{c} + \frac{1}{\hat{C}}\right)\right]$

Using the delta method and defining f as $\frac{t\hat{C}}{cT}\left(1 - \frac{1}{c} + \frac{1}{\hat{C}}\right)$, we find

$$\begin{aligned} f'\left(E(c)\right) &= \frac{df}{dc} \\ &= \frac{t\hat{C}}{c^2T} + \frac{2t\hat{C}}{c^3} - \frac{t}{c^2T}\Bigg|_{E(c)} \\ &= \frac{R(2-\theta-C\theta)}{C^2\theta^2}, \end{aligned}$$

$$
\begin{aligned}
f'\left(E\left(\hat{C}\right)\right) &= \frac{df}{d\hat{C}} \\
&= \frac{t}{cT} - \frac{t}{c^2 T}\bigg|_{E(\hat{C})} \\
&= \frac{R(C\theta - 1)}{C^2\theta^2},
\end{aligned}
$$

and

$$
\begin{aligned}
f'\left(E(t)\right) &= \frac{df}{dt} \\
&= \frac{C^2}{c^2 T} + \frac{1}{cT}\bigg|_{E(t)} \\
&= \frac{C\theta - 1 + \theta}{C\theta^2 T}.
\end{aligned}
$$

Substituting these and $Var(t) = TR\theta(1 - R\theta)$ and $Var(c) = C\theta(1-\theta)$ into (A2) gives

$$
Var\left(\hat{R}_{2_{BC}}\right) = \frac{R^2(2-\theta-C\theta)^2(1-\theta) + (C\theta - 1 + \theta)^2 CR(1-R\theta)}{C^3\theta^3} + \frac{R^2(C\theta-1)^2}{C^2\theta^2}\left[CV\left(\hat{C}\middle|C\right)\right]^2.
$$